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A conference of educators was called to deal with the need for improved high school mathematics teaching. Specifically the conference considered the need for inservice education for teaching new improved programs, the kinds of programs now in progress, and the role of school administrators in implementing them. Included are the addresses and the reports of group work sessions.

The many new uses of mathematics today demand greater understanding of the structure of mathematics, and more emphasis is placed on symbolic representation of ideas and relationships and less on computations which can be done by machines. Recommendations for curriculum changes made by various groups are described. The results of a survey of universities are given which show that over half have some kind of program of in-service training. Different types of programs are listed, with specific examples. Recommendations deal with personnel, costs, and practices.

The conferees reported many ways in which administrations are helping teachers up-grade their professional competence. For example they are encouraging further education by urging teachers to take advantage of local programs, and by providing funds or encouraging local philanthropic and industrial organizations to do so.

The importance of the communication of mathematics educational aims to other disciplines is emphasized, and cooperation is urged of high school with both elementary and college level math programs.

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Inservice Education
OF HIGH SCHOOL
MATHEMATICS TEACHERS

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Inservice Education of HIGH SCHOOL MATHEMATICS TEACHERS

Report of a Conference
Under the Joint Auspices of the
U.S. Department of Health, Education, and Welfare
and the
National Council of Teachers of Mathematics
Washington, March 17–19, 1960

Report prepared by Kenneth E. Brown and Daniel W. Snader, Specialists for Mathematics

U.S. DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE
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Foreword

AFTER many congressional committee hearings, the 85th Congress concluded that the Nation's educational program as a whole had a serious imbalance. In fact, the Congress considered this imbalance to be of such magnitude as to affect the national security. Their deliberations resulted in the passage of the National Defense Education Act, to be administered by the U.S. Commissioner of Education.

One of the educational areas found by the Congress to be in need of strengthening, in order to help strengthen the national defense, was the area of mathematics. Many other national groups, in addition to the Congress, have realized the need for improving the school mathematics program. The National Council of Teachers of Mathematics, for example, has published yearbooks on the unifying themes in mathematics. The May 1959 Bulletin of the National Association of Secondary School Principals devoted itself to the theme, "New Developments in Secondary School Mathematics." The National Science Foundation has financed the School Mathematics Study Group in its preparation of sample textbooks for grades 7–12. Other national foundations, too, have financed the writing of experimental materials for school mathematics. Thus, a local school has many sources from which to select new materials for the improvement of its mathematics courses.

The key to the improvement of any courses in the school curriculum is obviously the teacher. In this connection, as regards the high school mathematics curriculum, certain questions arise: Do the teachers need inservice education for the new improved courses? What inservice education programs are now in progress? How are school administrators helping in these programs?

To seek information on these and kindred questions was the purpose of the conference called by the U.S. Office of Education at Washington, D.C., on March 17, 18, and 19, 1960. More than 50 persons participated in the conference and contributed to its success. This publication presents the formal papers that were given and also a summary of the work session deliberations and panel discussions. It is being published with the hope that persons interested in the improvement of school mathematics will find specific suggestions applicable to their own schools.

The National Council of Teachers of Mathematics shared in the responsibilities of planning and financing the conference and also will share in the credit for whatever educational results accrue. Gratitude is expressed to all who took part in the conference. Their names and identifications will be found in appendix F.

J. DAN HULL, *Director, Instruction, Organization, and Services Branch.*	E. GLENN FEATHERSTON, *Assistant Commissioner, Division of State and Local School Systems.*

Highlights of the Conference

SPACE MISSILES are only symbols of the great explosion of scientific knowledge of the 20th century. One of the most important factors contributing to this explosion is the revolutionary advance in both the development and the use of mathematics.

Not only are new requirements being placed on mathematics in the fields of physics, chemistry, and engineering; but in other fields mathematics is being put to new and even more astonishing uses.

The biologist is applying mathematical theory to the study of inheritance; industry is using mathematics in scheduling production and distribution; the social scientist is using ideas from modern statistics; the psychologist is using mathematics of game theory. In fact, the logic of mathematical models shows promise as the basis for developing teaching machines for all areas of knowledge. The new uses of mathematics require less manipulation of formulas and equations, but greater understanding of the structure of mathematics and mathematical systems. There is less emphasis on human computation that can be done by machines, and more emphasis on the contruction of mathematical models and symbolic representation of ideas and relationships. Because of these new uses, mathematics is being firmly woven into the fabric of the national culture. The role of mathematics is not only to grind out answers to engineering problems, but to produce mathematical models (prototypes) that forecast the outcome of social trends and even the behavioral changes of a group. Such important new uses and interpretations of mathematics require that students have a program with a greater depth than the classical program designed for 19th-century education. The demands of society require a thorough revision of our present secondary school mathematics curriculum.

Projects Now Underway

The School Mathematics Study Group (*SMSG*).—For the first time in the history of education in the United States more than a hundred college mathematicians and secondary school teachers together have planned and written sample textbooks for grades 7 to 12. These textbooks have been tried out in many types of schools under competent teachers especially trained for the new task. The project was financed by the National Science Foundation.

The textbooks may be purchased from the director of the project, E. G. Begle, Drawer 2502A, Yale Station, New Haven, Conn.

The University of Illinois Committee on School Mathematics (UICSM).—Since 1951, the committee has carried on a strong program of study and experimentation in the improvement of school mathematics. Many schools have tried out the instructional materials and have thereby assisted in their development. Textbooks and teachers manuals for the first 2 years of the UICSM curriculum may be purchased from the University of Illinois Press, Urbana. These materials include the subject matter found in first-year algebra and plane geometry.

The Commission on Mathematics (CEEB).—This commission of the College Entrance Examination Board, established in 1955 and composed of mathematicians and high school teachers, has made recommendations for major improvements in the secondary mathematics curriculum. The Commission has issued publications which may be had by writing to the College Entrance Examination Board, Educational Testing Service, Box 592, Princeton, N.J.

The University of Maryland Mathematics Project (UMMP).—The project has thus far concentrated on mathematics in the junior high school. Published materials may be purchased from Dr. John R. Mayor, University of Maryland, College Park, Md.

Ball State Teachers College Mathematics Program.—This experimental program provides new material in algebra and geometry. Published materials can be obtained by addressing Dr. Charles Brumfiel, Ball State Teachers College, Muncie, Ind.

Boston College Series.—Under the supervision of the Reverend Stanley Bezuszka, S.J., materials are being planned for grades 8–12 in the Boston College Mathematics Series. The emphasis is on the structure of mathematics approached from the historical point of view. Inquiries about the program should be sent to Reverend Bezuszka.

Secondary School Curriculum Committee.—The committee, under the direction of Frank B. Allen, was appointed by the National Council of Teachers of Mathematics to study the mathematics curriculum and instruction in secondary schools with relation to the needs of contemporary society. The committee report, entitled "The Secondary Mathematics Curriculum," and special reports such as "The Supervisor of Mathematics: His Role in the Development of Mathematics Instruction," may be purchased from the National Council of Teachers of Mathematics.

Other Projects.—In addition to the projects specifically identified above, many local and State experiments and studies are underway. For example, a survey of a randomly selected sample of one-fifth of the public secondary schools revealed that 40 percent of the schools' administrators were planning to revise the mathematics curriculum.

Recommended Changes

Changes recommended by the various groups would—

1. Include the elementary concepts and language of sets.
2. Teach a more refined concept of function.
3. Strengthen the logical development of geometry.
4. Present in plane geometry some of the elements of analytic geometry and solid geometry.
5. Modernize the vocabulary of elementary algebra.
6. Study inequations as well as equations.
7. Stress understanding rather than manipulation.
8. Cultivate an understanding and appreciation of the structure of mathematics.

Teachers' Need for Inservice Education

Many educators are realizing that inservice education is a continuing need for all teachers if they are to remain abreast of new developments in subject matter and teaching methods.

For example, the School Mathematics Study Group realized that high school teachers in the SMSG experimental centers needed specific training for teaching topics newly incorporated into high school curriculums. A college teacher was assigned to work with the teachers once a week.

The University of Illinois Committee on School Mathematics attaches so much importance to inservice education that up until the time its materials were released (1959), it did not permit any teacher to participate in its project until he had spent from 2 to 8 weeks in preparation under its supervision (on various campuses). While the instructional material was being experimented with, supervisors were assigned to visit participating teachers during the school year and to assist them in the new type of teaching.

Current Programs

A questionnaire sent by one of the conferees to more than 100 of the large colleges indicated that 62 percent were giving inservice education courses on the campus and 40 percent in various adjacent school systems. Only 13 percent of the college institutes were paid for by the college and were free to the teacher. These institutes were of short duration, like the 1-day institutes at Northwestern University, University of Illinois, and Western Washington College of Education. Institutes for the reeducation of mathematics teachers have been sponsored by foundations and industries, including the National Science Foundation, the Ford Foundation, Shell Oil Co., General Electric Co., and the Camille and Henry Dreyfus Fund.

More than half of the colleges indicated that they did provide consultative help to teachers upon request.

More than half of the States had held conferences on the inservice education of mathematics teachers. About the same percentage have published brochures or booklets.

In about half of the 121 large schools studied groups of mathematics teachers were organized to study the mathematics program. TV programs at Buffalo, New York City, Denver, and Chicago have been used for teacher inservice education.

About one-third of the schools had organized an inservice education course which consisted of a college level class to introduce the new concepts in mathematics to the teachers.

Reasons for Changes in the Mathematics Curriculum

1. The subject matter of mathematics is constantly growing, not only in the area of advanced mathematics, but also in the area of elementary mathematics.
2. Mathematics today is being called on to meet a wide variety of needs which we had not dreamed of a few years ago.
3. The emphasis in mathematics is changing significantly: It is moving away from human computation to an understanding and construction of symbolic representation of factors that relate to scientific or social situations.
4. New mathematical ideas, language, and symbolism are being introduced to give better understanding of the subject.
5. There is growing realization of the need for better articulation between secondary mathematics and college mathematics.
6. There is a new awareness that some of the mathematics now being taught in high schools is obsolete and should be replaced by more significant subject matter.
7. A number of educational experiments have demonstrated the feasibility and advantages of teaching new topics in high school mathematics courses.
8. Several national groups of educators have made detailed studies of possible curriculum changes, giving specific recommendations.

Help From School Administrators

The conferees reported many ways in which the secondary school administrators are helping teachers to upgrade their professional competence. Some of these ways are briefly described below:

1. Encourage teachers to read and study books on modern mathematics. *Examples:* One school provided a list of such books which could be obtained through a simple telephone request. Niagara Falls Public Schools, N.Y., provides a professional library for teachers and encourages them to read and study the new books.

2. Develop a cooperative inservice education program between the public schools and a college or university in the area. *Example:* The Dallas Public School System has such a cooperative arrangement with the Southern Methodist University. The public schools of Syracuse, N.Y., and Syracuse University are also cooperating in this manner.
3. Provide the necessary funds to pay teachers to work on curriculum materials during the summer months. *Example:* The Eugene, Oreg., public school system is operating such a program.
4. Encourage teachers to visit classes in schools where the new materials in mathematics are being taught. Substitute teachers should be provided by the administration. *Example:* The Tucson, Ariz., public school system is providing this kind of encouragement and assistance to its teachers.
5. Set up salary schedules which provide appropriate increases for teachers who complete additional college courses in the subject matter of their teaching field.
6. Make provisions in the regular school budget for financing a continuous inservice education program.
7. Arrange the school schedule to allow teachers, *on released time*, to participate in seminars, curriculum study groups, professional organization meetings, and teacher inservice programs. One administrator of a large school system emphasized this point by stating: "It is our feeling that to reach all the mathematics teachers, we will have to provide inservice education on school time."
8. Encourage local industrial organizations and philanthropic agencies to sponsor inservice educational programs. *Examples:* The Timken Roller Bearing Co. at Canton, Ohio, paid 50 percent of the expenses incurred by local mathematics teachers in attending summer sessions at college. The local industries of Dayton, Ohio, paid for an 8-week institute for local teachers.
9. Encourage mathematics teachers to join their professional associations, both in mathematics and in general education.
10. Assist teachers to develop professionally by encouraging them to take an active part in experimental mathematics programs.
11. Organize a mathematics curriculum study committee consisting of teachers from various grade levels. This would help to provide continuous development of the basic unifying concepts of mathematics.
12. Engage well-qualified consultants to assist the local mathematics curriculum committee in their study of the total school mathematics program.
13. Provide a teachers' reading and conference room with books on modern mathematics and the new course materials.
14. Encourage the development of appropriate correspondence courses in mathematics for teachers. Each State department of education may assist one or more of the colleges in the State to develop such courses cooperatively.

Addresses

Keynote Address
Problems Address
Program Address

Although the conference sponsors are responsible for some necessary condensing of the following three addresses, the conclusions and interpretations are those of the authors and do not necessarily represent the views of the conference sponsors.

Needed Emphases in the Teaching of Mathematics and Implications for Teacher Reeducation

Dr. W. L. DUREN, JR.

HIGHLIGHTS OF THE ADDRESS

1. Considering the Nation as a whole, high schools are not staffed with teachers who are adequately prepared to teach mathematics.

2. If qualified teachers were available, it would be desirable for high schools to teach advanced (college) algebra, analytic geometry, calculus, sets, matrix algebra, and statistical methods and to stress disciplined mathematical reasoning.

3. Suggested ways for increasing the number of qualified teachers:

   Raise salaries.
   Raise qualification requirements.
   Make summer and academic year institutes more generally available.
   Expand commuting-type inservice training facilities.
   Increase the capacity of undergraduate and graduate schools for teachers.
   Import teachers from Europe.
   Devise some new types of teacher-education programs.

4. Suggested ways for training teachers:

   Through foundation, State, and other aid, prepare student textbooks (and teacher editions) and workbooks that would be more theoretical, more rigorous, and more complete than current ones are.

   Provide local seminars in which teachers would work through the new textbook materials.

   Provide film and kinescope expositions of 11th- and 12th-year high school courses.

   Provide in each State a traveling supervisor who would consult with local teachers and organize small regional conferences.

   Provide home study courses for mastery of the new mathematics.

   Provide Skinner-type teaching machines to help teachers develop good techniques.

   Award degrees and certificates only on the basis of rigid examinations.

5. Any kind of inservice education program is likely to give most benefit to teachers in larger administrative units, and hence to teachers who usually have least need for improvement.

6. The lectures now being given by many institutes are at such an advanced level that they are not appropriate for high school teachers.

THE ADDRESS

Mathematics is recognized as that high school subject which contributes most to the education of a future engineer or scientist. In fact, if a student is not already good in mathematics before he leaves high school, college mathematics can do little to help him become either one or the other. True, it can add to the competence of a student who has already achieved some mathematical mastery, but it can retrieve for the engineering and science professions very few students who have failed to get a good mathematics education in high school. Thus it is that junior and senior high school teachers are perhaps the key persons in building the foundations for our national scientific and technological strength.

In the United States a student takes up the calculus for the first time around age 19—if he takes it up at all. In Europe, by contrast, a student begins the subject at age 16 or 17, and it is not unusual to find a bright one beginning it at age 14. To be sure, we try to teach the calculus to a larger percentage of our school population, but the fact remains that our theoretical scientists are delayed at least 2 years in getting to this essential tool subject.

Physicists who have been working in the Massachusetts Institute of Technology Physical Science Study Group have a feeling that university mathematicians in the United States do not as a whole favor the idea of having the calculus started in high school. The reason that these mathematicians don't want to see the calculus taught there, at this time, is that so few of the present high school mathematics teachers are competent to teach the subject. In many cases the latter would do more harm than good were they to attempt to teach it. On the other hand, the reason that the university physics teachers demand it is that they cannot advance the study of physics unless the students coming to them already have some knowledge of the calculus. Again, to keep our place in this world we must have a substantial number of our students beginning the serious study of physics in high school. Indeed, we probably have to expect them to do at least 2 years of high school physics. There is no reason to believe that the students lack intellectual capacity or lack interest. The main trouble is that they lack mathematical preparation.

We have come to one of those times when the curriculum must be cleaned up. We must remove the deadwood, keep the substantial body of old mathematics whose value is ageless, but express it in a more precise and more powerful language and effect intellectual economies in the way it is learned and handled so that we can introduce those new ideas which are more valuable than the deadwood we remove. It has always been the history of intellectual progress that this sort of acceleration by economizing and simplifying things has enabled the younger generations to cover the ever broadening sweep of knowledge without being genetically superior to their parents. But, in coming to this time of reorganization and simplification of the mathematics curriculum to permit the earlier study of tool subjects like the calculus and the introduction of relatively new subjects like matrix algebra, sets, probability and statistical inference, we find that we have been losing ground for 30 years. Despite their willingness to work, our school mathematics teachers do not have enough mathematical power to adapt to the demands of a new curriculum.

If we find it in the common interest to have high school teachers take on responsibility for teaching traditional subjects like college algebra, analytical trigonometry, analytical geometry, and the calculus (which for generations have resided at just the next step after high school), we find them eager to do so, but on the whole not well enough prepared to teach these subjects without going back to school themselves. If we ask them to teach a more rigorous algebra, to observe the discipline of mathematical logic in their reasoning processes, to convert to the language of sets and set operations, to teach linear algebra or probability, they find these new ideas beyond them. If we ask them to accommodate to the new physics and chemistry or to use elementary dimensional analysis in a modern way, we find them unable to do that.

Yet, if we are to keep pace in the world and give adequate instruction to students of superior ability, we must have a corps of teachers who can teach, in the 11th and 12th grades, subjects which in the past have been largely the responsibility of the colleges. We need a special certification and a sufficient number of teachers so that each high school can have at least one of these advanced-level teachers who is capable of teaching superior students. The number now on hand is so small that we cannot yet advise high schools to undertake teaching the calculus for students to get advanced college placement, even though we know that this is an urgently needed offering for our future physical scientists and mathematicians. It is difficult to estimate the number of these teachers that we actually need, but when that number is found, the number that we should prepare should be multiplied by 5 or 10, since the losses to college teaching and to industry will be very great.

National surveys of the qualifications of mathematics teachers reveal two very distinct categories of these teachers. There is a fairly large group, constituting perhaps a third of the total, who at least on paper are well prepared. But at the other end of the scale there is another large group, also constituting perhaps a third of the total, who are poorly qualified. We may be reasonably sure that the rural and smalltown schools have more than their proportionate share of this poorly qualified third, even though they educate a large percentage of the student population. Schools in the socially decadent areas of large cities also probably have a share of the poorly qualified mathematics teachers, since in those areas juvenile delinquency makes life intolerable for a teacher well enough qualified to find a job in more pleasant surroundings.

Now, if we examine the means at our disposal to increase the number of qualified teachers and to improve the qualifications of existing teachers we find that our present ways of doing so merely tend to increase the qualifications of the already well qualified top third and to increase the number of qualified teachers in large urban schools without doing much for the small schools. Among the things which we might try to do are the following: (1) Raise teachers' salaries, (2) conduct more institutes—both summer and academic year, (3) expand the conventional commuting type of inservice training (where either the teacher commutes to the students or the students commute to the teacher), (4) increase the capacity of undergraduate and graduate schools for teachers, (5) import European teachers, and (6) invent some new kind of training program for teachers.

In the absence of any adequate scientific data on the effectiveness of these possible measures we can only extrapolate from experience. It seems reasonable to think that most of these measures would help the urban areas, the large schools, and the already well qualified teachers rather than the rural areas, the small schools, or the poorly qualified teachers. Thus, what we have done and what we are likely to do tends to separate still further the top third of our teachers from the bottom third. For example, in the large urban schools salaries are usually already higher and easier to raise than in the small rural schools, a raise in certification requirements could more easily be met, and teachers can much more easily be spared to take leave for academic-year institutes. There are usually one or more colleges within the commuting area where inservice courses could easily be offered. These same colleges could expand their teacher-training capacities and serve as centers of employment for those among their graduates who live in the area and who want to teach there.

But when a school board in a small town having no college of its own seeks a teacher from some college, agency, or another school out of town there are difficulties which make the task almost prohibitive. Frequently, bidding is entirely on a basis of salary. It is hard to get proper references. It is expensive and the result is likely to be unstable. The difficult search, choice, and persuasion in this market of extreme scarcity of teachers may have to be done all over again next year. It is much easier to pick up locally some married woman whose children are in school. Often the superintendent convinces himself that she has adequate qualifications, that she can be employed for a lower salary than can an outsider, and that she will stay. It is this type of "home folks" appointment which makes up a large component of the lowest third in qualification. Moreover, this process has been going on for a long time and some of these poorly qualified hometown teachers may no longer be young. Their family responsibilities (about three-fourths are married) make it difficult for them to go to academic-year or summer institutes. Also, teachers in a small school cannot easily be granted leave. Raising their salaries would probably not make better teachers of them, however justly the raise may be deserved. To improve their qualifications, inservice reeducation in some cases has been attempted with college professors commuting great distances to meet their classes. This is extremely wasteful of college manpower. College teachers have been willing to undertake this kind of travel in the recent spirit of emergency, but it cannot last and at best it is extremely limited in application.

Before returning to propose some measures for the betterment of hometown teachers, and thus for mathematics, in the small rural schools, I should like to comment upon teachers' salaries and certification requirements. It seems sure that the only long-term solution of the problem of providing American youth with adequate mathematical teaching is to raise salaries and raise certification requirements. We must pay enough to get good people and we have to demand that we get what we pay for. The fact that this is the direction of real solution is something we must not lose sight of while we are exploiting temporary measures to cope with the immediate problem. Probably a set of graded certification requirements with accompanying graded salary incentives is called for: A second-class certificate like the present minimum certification, a first-class certificate for normal high school work, and an advanced certificate for teaching the calculus or other advanced-standing subjects.

Let me also say something about those existing institutes and other fellowship programs which select the best applicants, often repeatedly, and offer rather high-level mathematics lectures sometimes inappropriate for the ordinary high school teacher. Many of the selected applicants have had the benefit of good training in the first place. Congress has put very severe limits on the programs in these institutes for preparation of college teachers; and our graduate schools cannot supply the demands of industry, universities, and National Science Foundation postdoctoral research fellowships for fully prepared mathematicians. This leaves the colleges short of qualified mathematics teachers and, as the community colleges spring up to take care of increasing numbers of college students, the problem will get much worse. In recruiting faculty, colleges will look more and more to persons who have received substantial mathematical preparation in the institute programs. This is not so bad except that it will rob the high schools of the best qualified teachers, and these schools surely need them.

Let us try to put together a complex of devices in an inservice program to meet the requirements of improving the qualifications of the hometown teachers. The pattern that we construct may be new in its total structure, though the elements will be familiar.

1. I propose that we prepare for each high school mathematics course a student's text (together with a teacher's edition)—more rigorous, more complete, more theoretical. The School Mathematics Study Group is doing this now, though I believe the form should be somewhat different. We will need workbooks, too.

2. Let us provide local seminars in which the teachers, including preferably at least one experienced teacher, will register and meet regularly to work through the text material.

3. The local seminar should carry some form of credit and suitable economic rewards.

4. As a supplementary form of instruction we should provide, on film or television kinescopes, a series of expositions or full courses for the 11th and 12th grades so that the presentations could be viewed jointly by students and teachers. Since these film lectures would not be longer than 30 minutes, time would be left for discussion at the end of the class period.

5. A traveling supervisor in each State should visit from time to time to receive a report of progress and keep the program going. This might be accomplished by means of small regional meetings rather than individual visits.

6. Home study courses by correspondence from university centers should be constructed to fit the new materials precisely. The weaker the mathematical preparation of a teacher the less he can adapt it to his own classroom activities.

7. Possible use of Skinner-type teaching machines to help the apprentice teachers develop techniques should be tried. Teachers are embarrassed to reveal their deficiencies in any open class and this is a serious barrier to their polishing their own technique through long practice, trial, and error. They might be less embarrassed to face a teaching machine. However, these machine devices can be expected to develop only routine technique, not mathematical reasoning.

8. Degrees and certification would be awarded only upon rigorous examinations conducted at university centers. The seminars, films, workbooks, and conferences would be regarded merely as aids to learning. Degrees would not be awarded upon the mere accumulation of credits.

9. Foundation, State, or other aid, to this inservice pattern would be to provide for writing the student's text and accompanying teacher's edition, making films, and supplying the traveling supervisor. Local

schools would be expected to relieve part of the loads of teachers taking inservice reeducation, particularly extracurricular loads, and provide additional stipends or salary increments upon the completion of the work.

The foregoing elements of an inservice reeducation program are adaptable to the large schools also. In fact, like all other devices we have been able to construct, they are probably more easily accomplished in the large schools.

Current Problems and Patterns of Inservice Education in High School Mathematics

DR. HENRY W. SYER

HIGHLIGHTS OF THE ADDRESS

1. This address is based on replies to a questionnaire sent by the author to colleges, State departments of education, and school systems.

2. The purposes of inservice education are to—
   - Improve teachers in content and in method.
   - Develop a unity of purpose in teaching.
   - Broaden the understandings of background and experience.
   - Keep administrators informed about the mathematics program.
   - Give teachers confidence as to the direction being taken in planned courses.
   - Keep teachers ready to investigate new ideas.
   - Keep the community informed about mathematics courses.
   - Help secondary teachers become informed about mathematics at the elementary level and the college level.

3. Three college-reported methods are the following:
   - Courses for persons already teaching.
   - Institutes financed by the National Science Foundation and by other means.
   - Consultative services by college staffs.

4. College-conducted inservice education originates both inside and outside the colleges.

5. State departments of education have reported these methods of inservice education as being used within their States:
   - Reading by individual teachers.
   - Groups of teachers meeting in informal discussion groups or in formal classes, with or without outside help.
   - Conferences and lectures.
   - Cooperative programs with nearby colleges.
   - Curriculum planning.

6. State departments of education have themselves used the following methods:
   - Conferences.
   - Publications.
   - Consultative services.
   - State curriculums.
   - Encouragement of liaison between colleges and school systems.

7. School systems have reported these methods for improving mathematical understanding:
   - Reading by individual teachers.
   - Groups of teachers meeting in informal discussion groups or in formal classes, with or without outside help.
   - Conferences and lectures.
   - Cooperative programs with nearby colleges.
   - Curriculum planning.
8. School systems have reported the following methods for improving mathematics teaching:
   - Conferences.
   - Visits by supervisors.
   - Formal courses.

THE ADDRESS

The purpose of my talk today is to present a summary and picture of the present situation of inservice education of mathematics teachers in the United States. This is a difficult undertaking because the United States presents great diversities in types of educational systems and schools, in the content of the mathematics taught, in the kinds of teachers, and in the ways whereby the schools, school districts, and States conduct their inservice education programs. In order to try to get a picture of all this, questionnaires were sent to various persons throughout the country. Most of my remarks will be based upon their replies. Although some of the answers were short and gave evidence that the people making them had not had enough time to be complete in their remarks, it is surprising how many answers contained 5, 6, 7, 8, 9, or even 10 typewritten pages of careful detail. I hereby express my appreciation and thanks to these persons for making it possible for me to present so many details.

You may be interested to know the breakdown of the groups receiving the questionnaire on inservice education for mathematics teachers. The group most likely to provide information by means of such a questionnaire seemed to be three: colleges, State departments of education, and school systems. The number of questionnaires mailed to individuals in these three groups, in the same order as just mentioned, was the following: 138, 67, and 212. The returns, percentagewise, were 66 percent from the colleges, 58 percent from the State departments of education, and also 58 percent from the school systems. I shall report the replies in this same order: Colleges, State departments of education, and school systems.

You may also be interested to know how the individuals were selected for receiving the questionnaire.

First, the individuals in *colleges:* The preliminary selection was composed of persons in colleges listed by the U.S. Office of Education in its *Education Directory, 1959–60, Part III: Higher Education*, under categories[1] III and IV, and *also* shown either as having a Dean of Education or as being accredited for teacher education.[2] Since this preliminary selection resulted in too many names, their number was reduced by eliminating approximately half the institutions in each State.

The names of individuals in *State departments of education* were obtained from the following sources: A list from the National Council of Teachers of Mathematics naming supervisors who devoted more than 50 percent of their time to supervising mathematics was used in its entirety. In this group were 40 persons in *State departments of education.* From another list of the Council giving the names of supervisors who devoted *less* than 50 percent of their time to supervising mathematics, five names were found for States not represented on the Council's first list, identified above. A third list (this one from the Office of Education) supplied 11 more names, representing States not covered by the Council's two lists. Finally, a group of 11 State departments of education remained where no one had been contacted for information. To them questionnaires were mailed addressed merely "Supervisor, Secondary Education."

The mathematics supervisors in *school systems* were reached by consulting two sources: (1) a list put out by the National Council of Teachers of Mathematics which identified 111 cities[3] which had a mathematics supervisor devoting more than 50 percent of his time to supervision; and (2) the Office of Education's *Education Directory, 1959–60, Part II: Counties and Cities.* From this directory were taken the names of all cities of more than 90,000 population and, in States having no city of that size, the names of the two largest cities.

Programs Reported by Colleges

Courses for Those Already Teaching

Courses for those already teaching were being given by 62 percent of the responding colleges, and 40 percent of these colleges were giving the courses in school systems—usually as extension work.

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[1] Categories designating institutions by highest level of training offered. For categories III and IV, the levels are as follows: *Category III—Only the bachelor's and/or first professional degree.* Includes those institutions offering courses of studies leading to the customary bachelor of arts or bachelor of science degree, and all those degrees which entitle the possessor to enter the profession indicated. *Category IV—Master's and/or second professional degree.* Includes those institutions offering the customary first graduate degree, and any degree earned in the same field after the first professional degree, or after a bachelor's degree in that field.

[2] The Directory indicates this fact by the symbol "ted."

[3] The word "city" denoted any community, city, town, or village appearing on that list.

A course for practising teachers has certain shortcomings. One of the most frequent seems to occur through misuse of the word "foundations." Originally, as applied to mathematics, the word had a fairly narrow meaning and it was a distinct part of the discipline of our subject. As happens with many words, however, this one was broadened to serve as a generally useful word, and consequently its meaning became blurred. For example, the following quotations from questionnaires returned by two colleges obviously do not retain the meaning originally intended by mathematics departments for the classification, "foundations of mathematics."

> Our course in foundations of mathematics includes such topics as short cuts and devices in multiplication, tests for divisibility, number puzzles, magic squares, and mathematical machines.
>
> A course in foundations of mathematics is not allowed for secondary education majors, but is approved for elementary education majors.

The meaning of the word "foundations" has certainly now become very vague. Still another college reported that in its courses to improve mathematics teaching the features that the teachers liked best were the teaching tricks. Other colleges said that the greatest handicap was that the teachers rebelled against the "hard study of mathematics." Obviously here are implied suggestions for the colleges in designing, naming, and describing their mathematics courses and in motivating teachers to understand the purpose of these courses.

Courses Attended by Mathematics Teachers and Others

This category in the questionnaire was intended to cover mathematics department courses attended by mathematics majors, psychology or economics majors who wanted mathematics courses, other majors, future mathematics teachers, and practising mathematics teachers. Since the inquiry concerned *inservice* education, such courses should not have been reported in any case where they were not attended by some teachers of mathematics. It is difficult to tell whether this is the interpretation given by the responding colleges, but 75 percent said that they offer such courses at the college and 7 percent said that they offer such courses in school systems. For example, the City College of New York City reported a cooperative program between the departments of education and mathematics.

Some of the shortcomings of the courses under discussion seem definitely to be connected with the matter of standards. For example: "We have 54 required quarter-hours for a mathematics major." However, this particular college sent along a catalog which shows that the highest level course in these 54 quarter-hours is one in the integral calculus. Or again: "We are now requiring a course in arithmetic content for elementary teachers before they receive their B.A." This news is not startling to those of us who have followed the need for training elementary and secondary teachers, but it is certainly a quotation to back up the feeling that the standards of mathematical content courses have been somewhat lax in the past.

One possible suggestion concerning courses for teachers and for persons intending to be teachers, would be to combine the lecture parts of the course, but have separate recitation or discussion sections for those in the class who are teachers, even if separate sections were not available for those not planning to be teachers. The sections for recitation or discussion could be used to discuss topics especially significant to teachers. Or, these sections could be used to help teachers who are trying to fill in their background, but who find that they are building upon mathematics learned so far in the past that they need a bit of special help to compete with others who are more proficient.

National Science Foundation Institutes

Certainly one of the most common and most profitable methods of inservice education for mathematics teachers in recent years has been the summer and academic-year institutes sponsored and paid for by the National Science Foundation. Sixty-four percent of the reporting colleges were scheduling these institutes at the time of the questionnaire. (The Ford Institutes and the Shell Institutes serve much the same purpose; and the Camille and Henry Dreyfus Fund Institutes, according to the reports from Montclair and Trenton State Colleges in New Jersey, also are working toward much the same goals.)

Comments on the questionnaires returned in the study reported here indicate that some colleges are beginning to depend too much on the National Science Foundation Institutes and are then excusing themselves for lack of effort in inservice education for mathematics teachers because of the fact that they have not been given a grant, from some source. Some of the colleges say they are doing nothing in this field because they have not been awarded a grant by the NSF.

Certainly the need will always exist for more local initiative and talent in designing, sponsoring, and conducting institutes. So much is to be gained from local development, sponsorship, and financing of inservice education programs that colleges should consider what responsibility they have for developing their own.

Another shortcoming reported is that some college programs include too wide a range of interest and abilities. This fact makes the programs difficult for instructors and unsatisfactory for participants.

What positive suggestions can be made concerning future NSF institutes? Here are a few in the paragraphs following.

There should be institutes designed for Statewide and citywide mathematics supervisors. Many of these supervisors are very new on the job—as recently as February 1960 or November 1959. They would appreciate help in improving their supervision, and would like to get together with other mathematics supervisors in order to share experiences and develop programs. Certainly, combining the less experienced with the more experienced supervisors would make an interesting and profitable 6-week institute.

Another suggestion would be to have a really rigorous evaluation of the institute program. Inevitably, in any group of individuals or projects some will always be better than others. Evaluations, however, can bring up the quality of the poor ones.

The final suggestion would be to try to eliminate many of the difficulties experienced by institute directors because of the large number of refusals by applicants after letters of acceptance have gone out to them from the institute. Refusals at this point mean that the institutes must continue to select applicants until the quota is full. One possible solution is that one central location should take care of all advertising, applications, and selections. The greatest fear resulting from this, of course, is that such a plan would lessen the power of the individual Institute director to make his own choice of applicants. This, however, need not be true at all, for selections could be made by a committee consisting of the directors or their representatives meeting centrally at one time. The applicants, of course, have applied only once to this central committee and have indicated, in order, their first three choices for institutes. By thought and preparation, most of the objections to this centralization can be removed, I am sure.

College-Financed Institutes Free to Teachers

Apparently college-financed institutes free to teachers are not very prevalent, since only 13 percent of the colleges answering the questionnaire reported having offered them. These institutes are naturally of short duration. Typical examples: The 1- or 1½-day conferences of the State University of Iowa, the twice-yearly institutes of Northwestern University, and the seminar on the materials of the University of Illinois Committee on School Mathematics of the Western Washington College of Education.

One of the reported shortcomings of these free institutes is that they try to be too broad, thereby wasting time. Another, that lectures without discussion seem to frustrate teachers who are used to participation. Still another, that discussion of research is neither interesting nor useful to teachers. It would seem that this last criticism might be qualified by noting that the manner in which a report of research is presented would certainly influence its reception by the teachers.

I have only two suggestions concerning these short-time free institutes. The first is that colleges should assume the expense to a limited extent, as a service to encourage and stimulate interest in mathematics teaching within a community. (This is especially true of State universities or State colleges.) The second suggestion is that the chief purpose of these institutes would be to give orientation or general information, since they can hardly serve as an intensive program to teach specific, limited topics in mathematics.

College-Sponsored Institutes Paid for by Teachers

The fact that this category was apparently interpreted in many ways makes one aware that the word "institute" does not have a clear meaning in the minds of many people. The meanings seem to include workshops and semester-length courses in some cases, so that it is very difficult to interpret the fact that 32 percent of the colleges reported sponsoring institutes paid for by the teachers. Some examples might be useful, however: The University of Florida held 3-week institutes in various counties during August; Northwestern University has had 1-day institutes charging 50 cents admission to the teachers; San Jose College has had a 1-week workshop; and other colleges listed many types of extension courses.

One of the chief obstacles to holding institutes seems to be the difficulty that teachers and college staff experience in finding time in their crowded lives to participate. Nevertheless, college sponsorship of such institutes (whether they be interpreted as 1-day or 8-week institutes) should be encouraged in order to avoid total dependence on national funds before local efforts are made.

Consulting Services Provided by Colleges

Forty-six percent of the reporting colleges have provided consulting services in mathematics to teachers and administrators coming to the college for help, and 55 percent have provided these services by sending consultants out to the schools. These two categories of course are not mutually exclusive. For example, Western Michigan University designates a day in the fall when new teachers come to hear and discuss solutions of problems peculiar to them. The extension division of Iowa State Teachers College regularly borrows teachers from the mathematics department to go out and do consulting in the schools. Rutgers has a program of "intervisitation" to mathematics classes at the college. (This probably means that local high school teachers have the opportunity to sit in on college mathematics classes.) Such procedures would certainly seem to be promising and should be looked into by other colleges.

Some of the shortcomings are summed up in this reply from one college: "Yes, teachers come to our college 1 day each spring on high school days." Such a regimented and artificial way of providing consultation service does not seem to be flexible enough. Allowing only 1 day a year when problems can be discussed has the same effect as postponing questions in a discussion group until the end of a 3-hour lecture. By that time most of the questions have been forgotten or do not seem important.

Another shortcoming of consultation services is the feeling implicit in many answers that a consultant who goes out to a school for only a single day achieves few permanent results. Or, as one person put it: "Consulting on teachers' workdays is useless."

The best suggestion that seems to grow out of the pattern of comments on consultation services is that long-term programs of cooperation should be developed between a particular college staff and a particular school staff. By getting to know each other, by helping each other work out solutions of the problems, and by finding the type of help that will be received readily, this association could grow into a most useful method of inservice education.

Origin of the Services: Within the College

Thirty-four percent of the answers said that the inservice training for mathematics teachers originated at the suggestion of college administrators; 52 percent said it was at the suggestion of individual professors. In general, the questionnaires were answered by individual professors (who unconsciously might have slanted the answers), but the proportions did seem to be in accordance with intuitive conclusions I had reached before receiving the answers. For example, Northwestern reported that a vice president had called a conference of schools in mathematics departments in 1945 to consider what should be done to help the mathematics program. San Diego State College indicated that the suggestion for applying for a NSF academic year institute grant had come from the dean of the graduate division.

The fallacy inherent in the expectation of having intiative for inservice education come from within a college is expressed in the answer from one college: "Our heavy teaching load in the mathematics department prevents our working with teachers." Even though we may feel that people will always find time to do what they believe is important to do, it is quite true that mathematics departments today have enough to do without thinking up a new project unless it becomes very important to them. The examples above suggest that colleges could well call together groups of professors and administrators who might be interested in promoting inservice education for mathematics teachers to have them discuss their responsibilities and possible plans.

Origin of the Services: Outside the College

Forty-three percent of the reporting colleges said that inservice education had been originated by administrators of school systems, 44 percent that it had been originated by teachers in school systems, and 18 percent that it had been originated by State departments of education. It may be true that this is not one of the major responsibilities of State departments of education and that therefore the small figure here should not be interpreted as a deficiency. For example, the University of California at Los Angeles reported that their courses for mathematics teachers had grown out of the needs and desires of teachers in their area. They also indicated that it is essential to include the administrators in the planning of these inservice programs. Courses at the University of Rochester have been supported by the State department of education.

With all this optimism we must note a type of shortcoming exemplified by the following words of one college: "The State department did not believe in subject matter. They are changing now and indirectly sponsoring a couple of workshops." It is hard to know what the State department did believe in if not in subject matter, but it probably felt that its time and efforts should be spent on general considerations rather than on consideration of the problems of each specific subject. One suggestion is that colleges prepare and circulate a bulletin to all school systems in their neighborhood, telling what they, the colleges, can do and what services the school systems can ask for. This might lead to cooperative efforts of several kinds planned together with neighboring school systems.

Programs Reported by State Departments of Education

Reading by Individual Teachers

Fifty-four percent of the reports from State departments of education checked reading by individual teachers. Funds available under title III of the National Defense Education Act were frequently cited as the financial source making such an activity possible. The publication of lists of books which would be purchased or the actual purchase of books to be circulated to school systems has been increased from these funds. The Georgia Department has had traveling displays of printed materials sent from meeting to meeting throughout the State.

A shortcoming that seems to be commonly found in the reports on this activity is in interpretation of "materials" to mean "equipment." Such an interpretation has been encouraged in the administration of the NDEA and is unfortunate. Perhaps, since the use of physical materials is less important in mathematics than in science or even in foreign language, there should be some change in the use of NDEA funds to permit a larger part of them to finance inservice education rather than certain physical materials. However, it is true that the purchase of books to be read and studied by pupils and by teachers will always continue to be important. One method to improve individual reading is for State departments of education to encourage local purchase of books and to help schools in deciding what to buy. Indeed, there is a feeling that the very best method of inservice education is to help individual teachers learn to read mathematics books and then study by themselves. They should not feel that they have a responsibility to go on with their study only when a formal group or class is formed. This is of course partly a matter of motivation, since anyone works better when he sees others in the same activity. It is also, however, a question of technique. Many teachers do not know how to get the most out of individual study of a mathematics book, and conferences and meetings of supervisors to improve the technique would be very valuable.

The State departments reported a variety of titles when naming the most helpful mathematics books, curriculums, and programs that they knew of within their States. (A list of these titles will be found in appendix A.) It is interesting to note that most of the titles seem to be on mathematical content. Two books seem to be in the elementary field and seven in the secondary. Only one book on method was named and that was in the elementary area (none in the secondary). Eight subjects were reported as curriculums or programs found helpful within the States. Eight items reported can be classified as miscellaneous. This last category includes films and also publications of the National Council of Teachers of Mathematics.

Study Groups (No Outside Help)

Fifty-nine percent of the answering State departments checked this activity. California commented that these self-initiated studies are the best kind. Various school systems in Minnesota have examined their curriculums by this means. A New Mexico mathematics teacher, returning from a University of Maryland summer institute, formed a study group. In Nevada, members of the mathematics departments meet often and take turns presenting topics to the whole group.

It must be remembered that such examples as the ones in this category—and all categories—are merely typical. They are not to be interpreted as the only ones or necessarily the best ones in the country. A shortcoming of this kind of activity, obvious to anyone who has worked with it, is the danger that such groups of friendly teachers working together will soon turn the session into one of telling anecdotes: Reporting how they like to teach a particular topic or how clever Johnny is. Such activities are relaxing but not very educational. Thus, a plan should be developed by any group having no outside help to assure that firm leadership be maintained.

Discussion Groups (Outside Leader)

Fifty-one percent of the answering State departments reported teachers' discussion groups that had an outside leader but no preparation expected from the group. In New Mexico this method was used to study textbooks being considered for adoption. Since textbooks are of obvious concern, teachers naturally would feel that such an approach is practical. It is valuable, however, only so far as it leads to a discussion of the mathematical concepts to be taught. Should discussion bog down on questions as to whether this or that textbook has the right amount of color in its illustrations or as to what types of exercises the teachers wish the authors would include, then the discussion might be interesting, but not particularly valuable.

New Mexico also reported worthwhile discussion resulting from a demonstration lesson on mathematics by means of a film.

A committee in Wyoming has a representative from each building within a school system, but this sounds more like a committee to establish policy rather than to provide inservice education.

Some of the shortcomings of discussion groups were brought out by the answers to a question as to what was the most useful mathematical topic discussed. Even with the word "mathematical" in the question, such answers as the following were reported: "Use of equipment such as tape recorder and overhead projector," or "proper use of manipulative devices and proper recognition of individual differences." Worthy as these discussions are, they do not answer the question concerning mathematical topics. Discussion groups should have variety, and the use of demonstration lessons, films, or TV programs to inspire discussion would certainly bring out a number of viewpoints. Naturally, it is important to restrict the range of the discussion in order to give unity, and this implies a firm chairman. It would seem that, instead of having a separate and distinct topic for each meeting, a group could restrict a series of meetings to a small range of topics in order to give unity and depth to the discussions.

Formal Classes

Forty-six percent of the respondent State departments of education reported formal mathematics classes with leaders from within the school system, and 46 percent reported these classes with leaders from outside the school system. (These 46 percent are not necessarily the same States nor mutually exclusive States.) The classes in Minnesota have come from local initiative and organization and there are plans for them in Pennsylvania. The New York, Oklahoma, and Pennsylvania Departments of Education have related formal classes with TV courses. Hawaii has formal workshop classes extending throughout the year. Delaware has its formal classes "tied to the salary schedule," which is certainly usual; but the classes are paid for by the school district, which may not be so usual.

One possible shortcoming of formal courses is revealed by the following comment from one State department of education: "One big problem is the paucity of resource people with the requisite characteristics to motivate the teachers to improve their instruction." This is an important criticism. How can we find enough college people with the correct background in mathematics *and* with a knowledge of teaching to staff very many formal courses for school systems?

Another comment from the departments is that teachers need some assurance that much of what they are already doing is right, but that in addition they must fit new ideas into their present curriculums.

Conferences and Lectures

Forty-six percent of the answering departments checked conferences and lectures. New Hampshire said that these were arranged by a group of teachers within the State and that also Saturday seminars were held at Dartmouth College, where teachers could discuss mathematics and methods of teaching. Historically speaking, such short conferences and lectures as a whole have often proved to be of little lasting help. Thus it seems that they should be used only occasionally as inspiration and that they could hardly be called a long-term method of "inservice education."

Curriculum Planning

Thirty-nine percent of the answering departments reported this activity. It is clear that well-conducted curriculum program of developing, writing, or studying a curriculum for a community or State can lead to much excellent inservice education. State curriculums should often be more general than they have been in the past; but the State departments of education should consciously help communities develop more specific curriculums applicable to separate counties, districts, or cities. However, many States still believe that a detailed statewide curriculum is desirable to standardize and improve the level of mathematics teaching.

State Curriculums

Thirty-six percent of the answering departments reported a State mathematics curriculum. Minnesota has a laboratory that attempts to evaluate curriculums to be used within the State. California is beginning a 3-year study of the curriculum in grades 1–14 of the State. Several States apologized that their curriculums are old and out of date. This may well be true, but I wonder whether we have not made them feel too guilty, since we have overemphasized the need for constantly revising the curriculum. It is hard to know what is meant by "old" and it may be that some are not so old as suggested. It should be carefully noted that providing State curriculums is not automatically a method for inservice education. Rather than the printed document, it is the active thinking through and writing of a curriculum and the interpreting of it in study groups that account for the inservice education.

Consultation Services

Fifty-two percent of the responding departments provide consultation services. Such services have many forms. Minnesota and Utah, for example, provides help to individual school systems as requested, and Maryland has helped to set up local workshops.

Conferences

Fifty-four percent of the answering State departments of education reported having set up conferences for mathematics teachers. Through its commission on mathematics, which holds many meetings and makes excellent reports, the Texas department has inspired many conferences. West Virginia has a dual staff for conferences: One person from mathematics and one from education to work as a team. Texas has a twice-yearly workshop in each of 13 districts, and Maine uses the traveling lecturer of the Mathematics Association of America as a nucleus around which to build conferences throughout the State.

Booklets or Bibliographies

Forty-five percent of the answering State departments reported publishing booklets or bibliographies. Newsletters, such as those in Minnesota and New Mexico, or lists of facilities and equipment similar to the one provided by Maryland are very useful. Bibliographies have been prepared by many States, including Connecticut, Iowa, Nevada, Oklahoma, and Utah. Oklahoma also has provided a booklet on the improvement of mathematics teaching that goes beyond the point of being merely a State curriculum.

The shortcomings of State department booklets and bibliographies are few, but it might be well to mention the overemphasis they sometimes place on physical equipment rather than on books or mathematical ideas. For example, one State department lists five publications available to the schools: Four of these are concerned with using equipment or buying equipment. The only suggestion about these publications is to study how they are being used and to *make sure that they are getting the proper results* rather than to assume that they will automatically get them.

Cooperative Programs With Nearby Colleges

Fifty-nine percent of the answering State departments reported this activity. In Georgia, university consultants are provided to the school systems and are paid for by the State department. Perhaps such consultation should not be in this category, which the questionnaire intended as covering long-range, continuing programs of cooperation combining high school teachers' experience and a college staff's somewhat broader knowledge of mathematics. However, there seems to be evidence that this meaning was not read into the question as stated. The 59 percent is therefore probably not very meaningful. In fact, the pattern of this type of help is so vaguely defined that this is a major deterrent to its use. It is therefore suggested that descriptions of such cooperative programs and the meaning of the term be developed, discussed, and written up by those communities and colleges now using them in order to help others who are planning them. *The Mathematics Teacher* might establish a section on inservice education, to which short articles could be sent by colleges, State departments of education, or school systems, sharing their good ideas with readers throughout the country.

Encouragement of Liaison With Colleges

Fifty-seven percent of the respondent State departments encourage liaison between school systems and colleges. In New Hampshire, a council composed of high schools and the University of New Hampshire has recently been formed to consider mutual activities at their respective levels. West Virginia has consistently tried to encourage the use of college consultants.

It is hard to know the shortcomings of liaison activities, since so few examples have been reported in the past, but it does seem as though this might be one of the fruitful sources of inservice education if time can be found for it in the schedules of the persons involved. Perhaps it should be pointed out that liaison activities could help the colleges by providing them with information about secondary school systems and thus bringing about a proper influence on the direction of the college curriculums.

Programs Reported by School Systems

Reading by Individual Teachers

Seventy-one of the answering schools reported this activity. The Minneapolis News Letter contains very helpful suggestions of books which teachers should read. A Bronx junior high schools in New York City reports that teachers are studying course outlines and following up with reading lists. Then they hold conferences within the school. Wichita, Kans., reports a traveling library. Niagara Falls, N.Y., says that single copies of books have been successfully placed in many school libraries (whether for students or for teachers is not clear). The proper use of books by high school mathematics students is not yet fully understood, and library resources are not utilized as well in that subject as in others. As far as teachers are concerned, the first activity may well be to locate books, review and make them available in the professional library, and give the telephone number where teachers may call and have the books delivered to their own classrooms. Certainly from an administrative standpoint no more can be asked. At this point the initiative of the teacher and his own motivation must take over in order to read the book and make use of it. However, supervisors and inservice education program directors do need to give help to the teachers as to how this reading can best be done.

As to books found most helpful, school systems named 5 *content* books at the elementary level and 27 at the secondary level. Six books on *methods* of mathematics teaching at the elementary level were selected and 1 book of this type at the secondary level. This book, interestingly, is "Mathematics for the Academically Talented Student", a methods book of a very specialized sort put out by the National Education Association.

This breakdown of mathematics books indicates that schools are thinking more about mathematics content than mathematics methods. But the answers also included 74 naming 15 publications which could be listed as "miscellaneous." Among these 15 were the yearbooks of the National Council of Teachers of Mathematics (the 24th in this series being named by 17 schools) and 2 periodicals, *The Mathematics Teacher* and *The Mathematics Student Journal.* (See appendix B for full list.)

Study Groups (No Outside Help)

Eighty-three percent of the answering school systems reported this activity. In Wichita, Kans., a citywide mathematics council is sponsoring such a group. Demonstration lessons followed by discussion are very helpful in Stockton, Calif., and also in San Diego, especially in the junior high school. Towson, Md., holds one meeting a week in a group to review calculus. Arlington, Va., is studying the school Mathematics Study Group materials. Glendale, Calif., has a 4-month workshop. Fort Wayne, Ind., has a junior high school committee studying various materials at that level. These are only examples and many more groups are working within school systems throughout the country.

One of the chief shortcomings seems to be the shibboleths developing where certain key words are offered in answer to all questions. Specifically, "Modern Mathematics" and "set" have occured many times as an easy answer to the question, "What are you discussing in these groups?" One way to improve the use of this method of inservice education might be to have demonstration classes of good teamwork in unled discussion. This means a demonstration class of teachers showing other teachers how a group working by itself without any specialized leadership can get most from an hour's working time. It might also be wise for a group to try to list the mathematical learnings which it expects will come out of the sessions and the books. (See appendix C for helpful sources of curricula and teaching materials reported by school systems.)

Discussion Groups (Outside Leader)

Forty-three percent of the schools in the study reported the leader coming from within the school systems, while 41 percent reported the leader coming from outside (with no preparation expected from the group). Spokane, Wash., and Wilmington, Del., had used the Mathematical Association of America lecturer to inspire the groups. Lincoln, Nebr., also reported conferences of this kind. Waterbury, Conn., mentioned using consultants provided by book companies. Racine, Wis., warned that there must be long-range planning in terms of a series of workshops rather than single-shot affairs. Corpus Christi, Tex., has had discussion groups 1 hour on alternate Tuesdays. Both Buffalo, N.Y., and Denver, Colo., mentioned TV programs for teachers which served as a basis for discussion groups.

Too often discussion groups try to cover too many different topics in a series of meetings. For example, one school system has a series of 10 meetings on 10 entirely different topics during the school year, hopping from such topics as number theory to non-Euclidean geometry, to matrix algebra, to algebra in the junior high school. Apparently the philosophy is that the variety will somehow make each teacher in the group happy. Instead, it leads to a disconnected, wasteful series of meetings.

Another shortcoming is the difficulty of reaching those seventh- and eighth-grade teachers who also teach English and social studies. Suggestions to improve the discussion groups may be numerous. It is obviously important to have a prepared leader and to keep the group small enough so that there can be real discussion. It is stimulating to base discussion upon demonstration classes, TV, or films, as previously noted. Finally, one may say that a series of meetings planned around a single topic would be more useful than a series made up of disconnected topics.

As to mathematical topics discussed by the groups having an outside leader (no teacher preparation expected), the breakdown is as follows: 17 schools reported 7 arithmetic topics; 22 schools, 11 algebra topics; 13 schools, 3 geometry topics; and 58 schools, 14 other topics. (See appendix D.) This last breakdown, "other topics," included advanced mathematics (such as coordinate geometry, probability and statistics, the calculus, digital computers) and also general background (such as sets, logic, variables, functions, relations, and finite mathematical systems).

It might be noted in passing that geometry does not seem to be of special interest to teachers: The only important discussion topic named by the schools was the possibility of integrating plane and solid geometry. To many of us, however, it seems as though geometry is one of the most urgent areas for discussion about the secondary school mathematics curriculum, for the future status of this subject is much more in doubt than that of many other subjects.

As to the most helpful mathematical topics taken up by the discussion groups, the breakdown is the following: 12 schools named 3 arithmetic topics; 9 schools, 6 algebra topics; 3 schools, 3 geometry topics; and 22 schools, 7 other topics. (See appendix E.)

Formal Classes

Twenty-seven percent of the schools in the study reported formal mathematics classes as being taught by teachers from within the school system; 30 percent, as being taught by teachers from outside the school system. Baltimore has an active summer program and provides voluminous and helpful notes to the participants. In Canton, Ohio, the Timken Roller Bearing Educational Fund pays 50 percent of the expense of all teachers attending summer school. Tulsa, Okla., has a workshop in June for its teachers. Newton, Mass., has classes for teachers conducted by the head of the department from

3:20 to 5 each Wednesday afternoon. Dayton, Ohio, has an 8-week institute paid for by local industries. New York City has produced a series of TV lessons in mathematics aimed specifically at teachers. This series is now being used by the State of Pennsylvania.

As to the shortcomings in the formal mathematics classes, here are some of those revealed by the study: Many course titles and descriptions are meaningless. This not only makes it difficult for teachers to know in advance what they would get in a course, but in addition it makes it difficult to report exactly what the affected school systems are offering. For example, a course called "advanced mathematics" is further explained as going all the way through analytical geometry. Standardizing course titles would be helpful. In that direction, it might be suggested that the use of TV lessons similar to those produced in New York City be extended to formal mathematics classes for teachers throughout the country.

Twenty-nine percent of the answering school systems reported formal courses for credit in methods of teaching given by someone within the system; 40 percent reported these courses given by someone outside the system. Chicago Teachers College has provided TV courses for Chicago teachers. The Cincinnati salary scale is tied to courses taken for credit.

One drawback to TV courses in methods of teaching is illustrated by the direction which a certain course eventually took. This course degenerated into a series of tricks for directing "the right answers." Perhaps the fault lay in the philosophy and background of the person presenting the course. At any rate, his suggestions for teaching mathematics were at least out of date.

Getting time for formal classes seems to be a problem. One school system said that its teachers turned down seminars because they assumed that one or two meetings a year were enough to keep them up to date in mathematics. Perhaps the solution to this problem lies in the following comment from one school system: "It is our feeling that to reach all the mathematics teachers we will have to provide inservice education during schooltime."

Conferences and Lectures

Fifty-seven percent of the schools in the study reported conferences and lectures on mathematics for the teachers. San Francisco stated that lecture demonstrations provide the most successful method of inservice education. Certainly it is a shortcoming of this method when it tries to cover too much material. For example, one 2-day institute announced that it would cover the following topics: Advanced algebra, non-Euclidean geometry, number theory, and the 10th-grade mathematics program. Could this not be cut down to 2 days of intensive application to any *one* of these topics? One school system commented in these words: "Most speakers present too many ideas and therefore there are no results in the classroom."

Conferences on Teaching Methods

School systems were queried about their attempts to improve the teaching methods of their teachers. The answers disclose that 47 percent hold conferences on the subject for beginning teachers only and 53 percent for both inexperienced and experienced teachers. Brentwood, N.Y., has two 9-day workshops for all elementary school teachers. Cincinnati, Ohio, has an orientation program for beginning teachers for the first half of the year. Washington, D.C., uses its local mathematics club as a center of conferences. Dade County, Fla., has brought consultants in for a day at a time. Doubt and shortcoming of this method are reflected in one school system's question: "How permanent are revival meetings? Such conferences may have little lasting effect." Other school systems suggest the use of follow-up materials compiled in as practical a form as lesson plans.

Visits Followed by Conferences

One of the most common and most useful methods of inservice education in teaching methods for mathematics teachers consists of visits by a supervisor followed by individual conferences. Seventy-three percent of the school systems in the study reported that such visits are made by people from within the school system, and 15 percent that these visits occur with someone from outside the school system coming to visit their teachers. Tucson, Ariz., has recently provided a substitute who takes the classes of junior high school teachers for half a day in order that these teachers may visit classes using materials from the School Mathematics Study Group or the University of Illinois Committee on School Mathematics and thus be able to lead discussions later. This is a valuable extension of the idea of supervisor visits. Roswell, N. Mex., has visitors to its classes from the State department of education. De Moines, Iowa, along with other cities, reports at least one supervisor visit to each teacher each semester. In Providence, R.I., good lessons seen by the supervisor are written up and circulated to all teachers.

So far as shortcomings go, one school believes that the practice of having a teacher-trainer who, in addition to the building supervisor, also works with beginning teachers, leads to an undesirable division of responsibility. Other schools believe that elementary and junior high school teachers who teach many subjects need more supervision but often do not get it, since supervisors are told to work only with teachers who are full-time in their own subjects. It must be noted that the intervisitation among classroom teachers may sometimes be more valuable than the usual visits by the supervisor.

Curriculum Planning

Sixty-one percent of the answering schools reported that their inservice education program is furthered by a curriculum-planning project. Riverside, Calif., has submitted a formal project toward this end. Both Dallas, Tex., and Fargo, N. Dak., as well as many other cities, have recently published excellent curriculums. Pittsburgh provides arithmetic-test kits as a valuable part of their inservice training. In Eugene, Oreg., teacher time is paid for in the summer for work on curriculum projects. Charleston, W. Va., is starting classes for able sixth-grade pupils and says the curriculum at other levels is thereby affected. Without supervision and coordination, curriculum-planning programs are impossible and so one shortcoming might be emphasized by this statement from a school system: "We have 150 teachers in our system, but no separate mathematics department. Coordination between the curriculum and inservice education is lacking." So far as curriculum planning goes, we might well suggest that it should be organized around mathematical concepts and not thought of as merely a collection of topics, magazines references to be read, teaching tricks, and test items.

Cooperative Programs With Nearby Colleges

Forty-three percent of the schools in the study reported this activity. Dallas, Tex., cooperates with both the University of Texas and Southern Methodist University; in Syracuse, N.Y., the Sloan plan has university professors teaching in the secondary schools. Albany, N.Y., cooperates with the nearby State college; Townsend, Md., has worked with The John Hopkins University; and Milwaukee, Wis., is attempting to plan a long-range program with the University of Wisconsin.

One of the shortcomings of cooperative programs is that many of the college instructors never go to see the high school classes at work. It is therefore necessary to convince such college instructors that close work with high school pupils is a part of their obligation. Can we also convince the teachers that regular, continued work with the college staff is going to be necessary?

Summary

The problems of inservice education can be put into two groups: The administrative and the procedural.

ADMINISTRATIVE PROBLEMS

Under the administrative problems five seem of great importance: Those of time, money, personnel, location, and supervision.

First, time must be found for the teachers to take courses and time for the supervisors or instructors to prepare them. The solution suggested here is that this time be made a permanent part of a planned program within a school or school system and that teachers and administrators accept the principle that inservice education shall continue indefinitely in the future and always be part of the work of the school and the teachers in it.

Second, the problem of money. Obviously, it costs something to provide books for teachers, salaries for instructors, or payment for preparing special TV programs. This fact will require local budgets to accept such a responsibility permanently.

Third, personnel. Both people to teach and people to attend these programs must be found. There are a few college people who have the right combination of interest and background in both content and methods to be able to appeal to the teachers. Using poor instructors has hurt inservice education programs in the past. Teachers who choose to take classes not required for every teacher in the system must be motivated to the hard intellectual activity involved. Not only their natural human inertia, but also their traditional practical viewpoint, must be overcome. Teachers must realize that although inservice education may be of no immediate application in the classroom, it may still be very valuable. The solution to the personnel problem may well be that active leadership must come more and more from those within the school system. People must be selected to teach with this purpose in mind. They must be educated by the school system and much of their regular time must be alloted for the work—this rather than that such responsibilities be given as an additional load. Also, these people must be provided with a budget to carry out their work.

Fourth, the location of courses and conferences. In many parts of the country the location must be considered carefully, since teachers cannot be expected to travel long distances to take courses and attend conferences. As long as such an activity is considered an emergency, teachers will put forth great effort, but if it is to be a permanent part of their re-education, the opportunity must be brought closer to them.

Fifth, the supervision of inservice education must be in the hands of a person trained in mathematics. Supervisors must not be nonspecialists and they must not share their time with other duties, as happens sometimes when a person who is given responsibility for a mathematics inservice program is one whose whole education and background are in other areas.

PROCEDURAL PROBLEMS

After the technical details of time, money, etc. have been taken care of, the person in charge of an inservice education program for teachers must decide what methods to use. Let use summarize very briefly 10 categories abstracted from all the foregoing details which apply to content and three which apply to methods of teaching. As far as increasing the background in mathematical content which teachers know, we have mentioned the following: Individual reading by teachers, study groups with no outside help, discussion groups with an outside leader, classes conducted for the teachers, conferences and lectures, cooperative programs with nearby colleges, published bulletins and bibliographies, consultation with the teachers, work on the curriculum, institutes, and workshops. With regard to methods of teaching it certainly seems as though general conferences will continue for motivation and inspiration. Also the following will doubtless continue: visits of supervisors to techers followed by individual conferences, intervisitation among teachers, and finally, but to a less extent, formal courses in methods of teaching provided within the school system.

Characteristics of a Desirable Inservice Education Program for High School Mathematics Teachers

DR. HENRY VAN ENGEN

HIGHLIGHTS OF THE ADDRESS

1. In mathematics we are experiencing a rejection of the traditional approach in favor of a modern one.
2. In order to prepare teachers for presentation of the new mathematics, the first efforts were directed only to teaching more subject matter. Those efforts are now somewhat discredited.
3. School administrators should provide leadership for modernizing mathematics instruction.
4. Supervisors should carry active responsibility for inservice education of teachers.
5. School system should provide on-the-job courses for its teachers. These courses should concentrate on the specific needs of the school curriculum.
6. Summer institutes and summer schools offer opportunities for inservice education.
7. Television and radio programs provide opportunities for inservice education in some communities.
8. Teachers need careful conditioning before being asked or permitted to teach the new mathematics.
9. School corporations should bear the cost of the teachers' inservice education.

THE ADDRESS

Archibald MacLeish opened a recent television play entitled "The Secret of Freedom" with the words, "Ours is a strange time in America." MacLeish was referring to political, economic, and religious thought in America. His words apply equally well, however, to many other phases of American life. The technological explosion that has taken place since World War II has caused us to reevaluate our usual way of life and to seek other, and still better, ways to preserve that which we hold even dearer than life. In many instances, brute facts have caused us to do things which are strange in the sense that they are not usual.

Problems in education have much in common with problems in Government or problems in family life. It is only in details that things are different. In mathematics and science we are experiencing a rejection of a traditional approach for one needed by the contemporary mathematician and scientist in order to work on pressing social problems. In view of the problems confronting our society, there can be no excuse for teaching mathematics in the schools unless it encourages our youth to think like mathematicians. Of course, a similar statement could be made about art, music, social science, science, and any other subject taught in the schools. Schools do have their utilitarian objectives, but above all they are working with the intellect of the young people and this intellect must be so developed as to enable today's youth to understand tomorrow's mathematics. This is a primary function of the schools.

Today's mathematics is somewhat different from yesterday's mathematics. Today there is a greater emphasis on solving problems in the large, a more intense search for related structures in various branches of mathematics, a desire to solve groups of problems rather than seek a solution for a particular instance. Since mathematics has changed during the past few decades, it goes without saying that the mathematics of our schools and colleges must change. In the schools, many staff members have been prepared to teach the old mathematics. Such preparation is not suitable for today's teachers. This raises a problem for those whose responsibility it is to provide leadership for an educational program. How does one encourage a staff to keep up to date, to rethink a mathematics program, and to build a new program?

The first reaction of many people who became concerned about mathematics in our schools was to teach the teachers more mathematics. As a result, institutes were established whose emphasis was on teaching mathematics to teachers. It soon became clear that some of these institutes were overshooting their mark. The advanced mathematics courses offered were not suitable for teachers and in many instances had little or no bearing on the mathematics taught in the schools. The institute courses bore no fruit in terms of the ultimate objective: to change the attitudes of the teacher and the way he teaches in the classroom. This does not mean that teachers should not be taught more mathematics. Indeed, more mathematics for the teacher is a necessary step toward improving teaching and the curriculum, but it is not sufficient. As evidence, one need only cite many recent college programs whose spirit was not much different from that of the high school programs. College staffs know more mathematics than high school staffs, but the spirit, and sometimes the content, was not much different from that offered in the high schools.

Now we come to an important question: What are some of the desirable characteristics of a good inservice program in mathematics for teachers of grades 1 through 12?

Leadership

The schools cannot improve either the curriculum or the inservice program without adequate leadership. Superintendants and principals set the intellectual atmosphere in which a staff must work, and this atmosphere will condition the attitudes, the working habits, and the instructional goals of the staff. No staff can do effective work in an educational atmosphere which is inimical to the spirit of education itself. Creating the proper intellectual atmosphere is one of the most important tasks of an administrator. It is particularly important in schools where the drive for academic excellence, social responsibility for education, and continued learning is not so strong as in colleges and universities.

Once having established the proper climate for study and learning, the administrator is faced with the task of selecting someone to do the detail work and provide specialized leadership. No inservice program can function effectively without adequate leadership from the supervisory staff. In positions of this type the schools need vision as well as adequately trained personnel. The supervisor must know mathematics thoroughly in addition to vital facts about children, curricula, teachers, administrators, and administration. The supervisor must know the school system and the mathematics staff with all of its strengths and weaknesses. Without a knowledge of mathematics he is in the position of leading a staff without a certain knowledge of where to go. The supervisor must be able to make independent judgments concerning the value of certain mathematical topics. These judgments cannot be made without an understanding of the nature of the subject itself.

Schools should not underestimate the amount of mathematical knowledge required of a supervisor of secondary mathematics. Although one should never underestimate the value of a knowledge of such things as basic principles of psychology and philosphy, and the basic techniques of supervision, it is also important not to underestimate the need for a knowledge of mathematics. All the values that one expects from the mathematics classroom rest on a sound mathematical program. Such a program cannot be developed by one who is insecure about the mathematical principles to be taught.

On-the-Job Courses

A program for the improvement of teaching and the development of a sound course of mathematical instruction will have as one of its components courses of instruction for which the teacher may enroll while teaching. These courses can be divided into two categories: those taken for credit toward an advanced degree and those taken to fulfill requirements for advancement in salary. Of course, these are not mutually exclusive categories.

Courses offered by collegiate institutions for credit toward an advanced degree are beyond the control of the school system, as they should be, and their basic purpose is to advance the knowledge of the teacher in his speciality.

Courses sponsored by the school itself are quite another matter. They should be highly practical in nature—practical in the sense that they discuss mathematical ideas that have their roots in the courses being taught to the children. These are the courses that can teach the same algebra in much the same way that it should be taught in the 9th grade, for example. However, there is one important difference. Presently, the spirit and the content of this course is undergoing a change. Perhaps it would be safer to say that the change is being strongly recommended. The course in first-year algebra should be taught to the teachers with the spirit and the organization of ideas demanded by present-day mathematics. It should stress fundamental ideas and follow these ideas through the 12 school grades in order to give the teachers a better grasp of how certain ideas pervade mathematics. Too frequently such fundamental concepts as functions, strategies of proof, relations, transformations, deductive systems, the real number system, and other basic mathematical ideas are too vaguely understood by the teachers. These are the ideas that the on-the-job courses should cover thoroughly. They should be discussed in the light of contemporary mathematics, but always keeping in mind how they can best be taught to the elementary or high school pupil.

In 1908 Felix Klein published his famous book, "Elementary Mathematics From an Advanced Standpoint." Using this title as a guide, one could best describe the spirit of the on-the-job courses for teachers by calling them "Advanced Mathematics From an Elementary Standpoint." Or perhaps "Elementary Mathematics From a Contemporary Standpoint" might be better. Whatever the title, the basic motivation is clear. These courses must help the teachers reorganize and modernize the mathematics that they teach. Also, they must provide the proper mathematical spirit from which good instruction stems. Granted that teachers have the time to take such courses, there are few better ways to improve the teaching in the schools.

It would be well to dwell for a bit on the problems of the elementary school regarding inservice courses. There is no greater need in the educational system than the need to provide an adequate mathematical background for the elementary teacher. The colleges have been seriously negligent in this matter. The schools must teach, on an inservice basis, that which the colleges should have taught. Many elementary school teachers have obtained their teaching certificates without taking any mathematics beyond that required by the high school. As a result, the mathematics program in the elementary school is suffering. It is largely conceived of as a computational subject taught in order to enable our future citizens to check the grocery account or to see that the bank balance is correct. It seems hardly possible to find a better means to kill the spirit of mathematics. To remedy this situation, the elementary teacher must be introduced to modern arithmetic, certain fundamental ideas of geometry, number theory, and algebra. For the present, the schools must do this, and it must be done through inservice courses.

The problem of staffing inservice courses can be difficult. In some cases nearby college staffs can be used. However, the college instructors who can teach the kind of practical course described in this paper are few and far between. It is difficult for anyone to teach a good course for teachers without having had considerable experience teaching school mathematics. It is in this role that an adequately trained supervisor can be of great service to the system. A supervisor whose leadership role is recognized and whose knowledge of both mathematics and the classroom is respected by the teachers can be worth his weight in gold to any system trying to improve the mathematics taught in its classrooms. Too few supervisors can perform this function for the schools.

Summer Institutes and Schools

Using vacation time for attending summer school is a well known means for teachers to get inservice education. Mathematics teachers have many opportunities for their summer's educational activities to be financed through attending institutes sponsored by the National Science Foundation, or by various companies such as the Shell Oil Co.

If the schools are to get the full value for money invested in advanced education, some control must be placed on the kind of courses that may be used for credit. It is common practice for teachers to acquire a master's degree without taking any courses in the field of their major teaching assignment. Most certainly, this is not a desirable situation. Mathematics teachers should not be given an advanced place on the salary scale for a degree earned outside the field of mathematics. While it is true that many of the courses taken to earn a nonmathematics degree are valuable, yet it can be hardly be imagined that a concentration of courses in administration or supervision could be entirely fruitful in producing a better mathematics teacher. It is entirely reasonable and proper to require that mathematics teachers take at least one-half of their courses for salary advancement in mathematics. For elementary teachers, schools must require that courses be taken in various academic departments in order to strengthen the academic backgrounds of the teacher. Most certainly, the real teacher is the teacher who is learning. The teacher who has stopped learning more about the subject taught is the teacher who has ceased to be effective in the teaching situation.

Television and Radio Programs

For some systems, it is entirely feasible to use television and radio in an effort to reach large numbers of teachers. The possibilities of television have not been entirely explored in the inservice education of teachers. Frequently, television is used as a means to teach more children with fewer teachers. It might well be that television can be used effectively to produce a change in the mathematics taught in the schools.

The schools have a serious problem in obtaining well-qualified supervisory personnel for the schools. The task of obtaining well-qualified teaching personnel for telecasts is about equally great. The telecast teacher must be well acquainted with the newer trends in mathematics, and furthermore he must be able to adapt the newer mathematics to classroom use. This is a large order for the schools. We desperately need people who have a command of mathematics and who are master teachers to demonstrate teaching methods and the newer mathematics for our schools. Television has great possibilities for improving the practices of the schools but it must be used wisely and with the proper teachers in charge.

Boundary Conditions for Inservice Education

While working with a group of people for fundamental changes one must keep in mind that certain boundary conditions must be satisfied if an adequate solution is to be achieved. It would be well for any supervisory staff to keep these conditions in mind.

In the first place, the educational background of the staff at time $t=t_0$ must be given due weight. In many school systems the teachers have taught the older type of mathematics for many years. The newer courses demand not only a change in organization but also a change in method and content. Introducing new courses into the curriculum without some thought as to the change in spirit may consist of only a change in name and not a change in fact. Schools contemplating a change in courses should make adequate plans for the change by establishing an intensive inservice program. This program should give the teachers the needed background for the new courses and should supply the mathematical and psychological reasons for the change. Without a carefully planned program, the introduction of new material into the classroom might well end in disaster and prevent subsequent change.

In the second place, the supervisory staff must take into account the rate at which changes can be brought about without creating educational chaos. Forcing changes may endanger the whole move for reform. It might be well to introduce changes in a very limited way during the first few years. In such cases the inservice courses should be closely linked to the changes being initiated in the curriculum. For example, some schools might introduce reforms in first-year algebra without making any changes in senior high school mathematics. Other schools might have a staff capable of absorbing changes at a more rapid pace. The supervisory staff and its advisory committees must estimate the rate at which changes can be brought about.

On the other side of the coin, the supervisory staff and the administrative staff must exercise their leadership functions. Changes and efforts to produce changes cannot be left to a vote of the teaching group. Leadership implies a responsibility to present issues to the teachers. Leaders cannot wait for the drive for reform to come from the teachers. They must continually create conditions that tend to move the staff forward. Because of these leadership obligations, the supervisory staff must be strong.

Schools must make haste slowly, but they must make haste. To make haste the schools have a responsibility for providing conditions which are favorable for making haste.

Maximizing Inservice Efforts

In this day and age very little of value is attained without spending money and, unfortunately, sizeable sums of money are frequently required to get the things needed to provide the best educational program for our youth. To provide a good program in mathematics and science for the pupils in our elementary and secondary schools will cost money. It may even cost the schools as much money as they spend for driver-education programs, or even as much money as they now spend to field 11 of their best athletes each week on the football field. Under present conditions, there is great danger that the schools will expect teachers to reeducate themselves at no cost to the taxpayer. This is not a feasible position to hold.

In view of our present situation, schools must break with the traditional patterns of teacher education. Four years of college education with a sound mathematics major obtained 20 or 30 years ago is not a secure basis on which to embark on a new mathematics program. Furthermore, many of the very recent college graduates have been trained in rather traditional freshmen and sophomore college mathematics courses. These teachers need courses not now provided by many educational institutions. There is only one answer. The schools must provide the courses at their own expense.

The schools have been demanding that teachers perform an inhuman task. They expect a teacher to teach five or six classes a day and take care of many extracurricular activities in their "spare" moments. Under these conditions, it is almost humanly impossible for teachers to stay abreast of the times in secondary mathematics. Because of the traditional workload for teachers and some outmoded concepts of teacher education, the schools have found themselves confronted by a demand for reform for which the teachers are not prepared. Administrative techniques must be further developed to prevent this impasse from happening again.

Those administrative techniques that have been found useful for reeducating teachers can be enumerated. For the most part, they are well known devices for encouraging teachers to give more thought to classroom problems. They can be adopted with varying amounts of money needed to introduce them, or to support them, over a period of years. The following are some of the better known devices for inservice education:

1. Financing summer schools for teachers.
2. Financing inservice courses for teachers.
3. Providing released time for seminars, inservice courses, and educational meetings.
4. Strengthening the supervisory staff.
5. Making better use of educational television.
6. Encouraging teachers to apply for financial support to attend academic-year and summer institutes sponsored by various agencies.
7. Paying premium salaries to teachers who have advanced degrees in mathematics.
8. Requiring that advanced training to be taken in the teacher"s field of specialization.

Other measures for keeping the staff up to date can be developed also, since the foregoing list does not exhaust the possibilities. However, the one characteristic common to all methods is that they cost money. It should not be otherwise. Commercial and business concerns expect to spend money to educate their staffs. Schools should expect to do no less.

Conclusion

Present conditions relative to science and mathematics should serve as a lesson to the educational world. We have been lulled into an educational sleep thinking that mathematics will always be the mathematics of Euclid, and that Euclidean geometry will always hold the place of eminence it has held since the days of the Greeks. But the truth is that mathematicians have drastically changed their concept of mathematics during the past few decades and that, in a sense, they have changed their concept of how to attack mathematical problems. This changed concept has caught our schools and colleges in a deep curricular sleep. They were not anticipating the change and were not ready to cope with any change in their concept of the goals of mathematicians. As a result, staffs are not adequately prepared for the needed changes.

The lesson the educational world must learn from our present experience is that we must always be ready for a change in the mathematician's concept of his subject and, with the rapid rate of growth of the subject, it might well happen that in the next 50 or 100 years schools will again be called upon to bring their teaching up to new standards. Knowing that this is likely to occur, the schools must not be caught unprepared a second time. To avoid this, there can be but one answer, namely, adequate inservice education provisions for the teachers. In effect, the schools must provide schools for their teachers and make the necessary expenditures of money and time to encourage teachers to attend them. It has often been said that learning is a never-ending activity for the human being. Most certainly, the good teacher and the good teaching staff will never cease learning; in particular, they will never cease learning mathematics. It cannot be otherwise, for how can a teacher instill the love of learning if the teacher does not know what it means to love learning sufficiently to pursue it constantly?

Group Work Sessions

Promising Practices in Inservice Education of Mathematics Teachers

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Promising Practices in Small Schools and Small School Systems

Chairmen for the Session:
DR. MILTON W. BECKMAN
DR. JAMES H. ZANT

HIGHLIGHTS OF THE REPORT

1. Regional conferences of high school mathematics teachers should be organized in order to examine and discuss the evolving new content of the subject.
2. Some military officers who have strong backgrounds in mathematics might be prepared through inservice programs and engaged to teach in the public schools. The same applies to mothers whose children have grown up.
3. Teaching machines, tape-recorded lessons, and television-film lessons are useful teaching aids.
4. As a means for improving teachers' competence, they could be sent to graduate schools and teacher institutes, with the tuition paid by the teachers' school.
5. Local colleges and State departments of education should be called upon for assistance in formulating and conducting inservice education programs.
6. State-approved colleges and universities should be encouraged to establish appropriate correspondence courses to meet the particular needs of mathematics teachers who cannot attend campus classes.

INTRODUCTION TO THE REPORT

This discussion group decided to report according to the following plan:

1. Identify the problem situations which small schools face in establishing and conducting inservice programs for teachers of mathematics.
2. Suggest possible ways and means by which small schools can develop inservice mathematics programs and/or strengthen and improve those already in operation.

The problems on inservice education encountered by small schools or small school systems may also be encountered, to a lesser extent, by medium-size or large schools systems. Such matters as the lack of adequate funds, sufficient personnel, program flexibility, proximity to area colleges, and other factors are present to a greater degree in small schools than they appear to be in medium size or large schools. With this in mind, the group has focused on problems as they exist in small schools, and has given illustrations of possible solutions geared to problem situations in these schools. The problems identified, and the solutions suggested do not necessarily preclude some applicability to the larger schools.

THE REPORT

Problem Situation 1.—Lack of understanding by teachers and administrators that there is an imperative need for extensive reeducation and that additional content covering the newer concepts of mathematics must be learned.

Suggested Solutions.—The group's discussion on the first problem situation which they identified resulted in the following suggested solutions:

1. Identify knowledgeable persons in the field of mathematics to present the nature of the problem, data, and materials to mathematics teachers and other faculty members, including superintendents and principals.
2. Establish regional conferences sponsored by such organizations as State mathematics associations, National Council of Teachers of Mathematics, Mathematical Association of America, American Mathematical Society, State colleges and universities, and State departments of education, in order to examine and discuss the evolving new content in mathematics.
3. Promote programs which provide for the study of the evolving new courses in mathematics at professional institutes, State conventions, and meetings of school administrators.
4. Read and circulate among mathematics teachers the newsletters and pamphlets issued by groups now sponsoring curriculum revisions.
5. Provide mathematics teachers with adequate supplies and equipment for inservice programs.
6. Urge the establishment of a clearinghouse at the area college to provide services and faculty resources to assist school mathematics teachers in finding solutions to their professional problems. A college staff member should be assigned by the college to discharge this responsibility.

Problem Situation 2.—Absence of a well-defined mathematics curriculum K–12, designed approximately according to content and grade placement.

Suggested Solutions.—Five suggested solutions came out of the discussions on the second problem situation:

1. Reexamine local mathematics curriculums in light of current proposals by groups such as the Commission on Mathematics (College Entrance Examination Board, Box 592, Princeton, N.J.), University of Illinois Committee on School Mathematics (University of Illinois Press, Urbana), School Mathematics Study Group (Drawer 2502–A, Yale Station, New Haven, Conn.), University of Maryland Mathematics Program (College of Education, University of Maryland, College Park, Md.), and

others. Organize local, county, and State study groups making maximum use of college, State and other professional consultants.

2. For each secondary school, establish a professional mathematics library, including books and instructional materials such as those recommended by the groups above.
3. Encourage membership and participation in local, State, and National professional mathematics organizations.
4. Read and study the magazines and other publications of professional mathematics organizations, especially those published by the National Council of Teachers of Mathematics (1201–16th Street NW., Washington 6, D.C.).
5. Encourage informal interschool visitations of teachers, especially to schools where experimental projects in mathematics are underway.

Problem Situation 3.—The need for continuous anticipation and study of the emerging curriculum patterns in mathematics.

Suggested Solution.—The group arrived at one overall suggested solution for the third problem situation:

> Obtain evaluation reports of the pilot projects proposed by the groups named in Solution 1 of Problem Situation 2.

Problem Situation 4.—The inadequate preparation of many currently employed mathematics teachers not only for the traditional content of the subject, but also especially for the emerging content.

Suggested Solutions.—The group's five suggested solutions for the fourth problem situation follow below:

1. Encourage mathematics teachers to apply for scholarships to attend the academic-year and summer mathematics institutes of the National Science Foundation and those sponsored by private industries.
2. Read and study the 24th yearbook ("The Growth of Mathematical Ideas, Grades K–12") of The National Council of Teachers of Mathematics. (The council's 23d yearbook, "Insights Into Modern Mathematics," may be used as a supplement to the study of the 24th.)
3. Study, individually or in small groups, the materials recommended for a professional mathematics library in Solution 2 of Problem Situation 2.
4. Encourage teachers to attend summer sessions or inservice workshops organized by local groups and planned especially for mathematics teachers.
5. Consult the State department of education for information about the TV mathematics program "Continental Classroom."

Problem Situation 5.—The lack of instructors at college oncampus and offcampus centers.

Suggested Solutions.—The following suggested solutions for Problem Situation 5 represent the group's thinking:

1. Investigate the possibility of having visitations by representatives of the Mathematical Association of America under the High School Visiting Lecturer Program.
2. Encourage colleges and the State department of education to provide consultative services and visiting lecturers in mathematics.

3. **Encourage continued cooperation by the State department of education and the U.S. Office of Education in supplying consultative services.**
4. **Urge college administrators and heads of college mathematics departments to recognize that an emergency exists and to allow members of the mathematics departments to take on additional workloads in inservice programs, with appropriate compensation. (Intelligent discretion in taking any such action is of course recommended.)**

Problem Situation 6.—Insufficient personnel having the enthusiasm and dedication necessary to promote inservice education for mathematics teachers.

Suggested Solutions.—Five suggested solutions to the sixth problem situation emerged from the group's discussions:

1. **Encourage the State department of education and colleges to prepare and distribute to schools lists of competent personnel having enthusiasm and dedication, who would be available to promote inservice education for mathematics teachers.**
2. **Urge teachers and administrators to write to the State department of education for lists of competent advisory personnel.**
3. **Appeal to directors of college and university extension services for help in identifying problems in the mathematics curriculum at the local level, and invite their attention and help in promoting and sponsoring inservice education.**
4. **Utilize the talents of many people in supervisory ways for promoting inservice education.**
5. **Enlist the talents of supervisors who show dedication to the cause of inservice mathematics education.**

Problem Situation 7.—The need for reexamination and gradual upgrading for permanent certification of mathematics teachers.

Suggested Solutions.—The group suggested four solutions to meet the seventh problem situation:

1. **Promote cooperation among the State department of education, colleges and universities, teacher-education groups, and professional-standards groups for the purpose of meeting the problem.**
2. **Emphasize inservice education programs as a means by which mathematics teachers can become qualified for permanent certification.**
3. **Encourage permanently certified teachers to continue their study of the evolving new materials in mathematics.**
4. **Encourage colleges, during the current emergency, to assist in certification upgrading by providing credit courses particularly suited to the needs of mathematics teachers, even though these courses are not at the moment self-supporting.**

Problem Situation 8.—The less-than-optimum use of mathematics teachers.

Suggested Solution.—The discussion group had one general solution to offer for Problem Situation 8:

In this emergency, assign competent mathematics teachers to teach mathematics rather than assigning them to teach other subjects or to take on duties which might be handled equally well by other personnel.

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Problem Situation 9.—Unwise assignments of nonmathematics personnel to teaching mathematics.

Suggested Solution.—The group also had one general solution to Problem Situation 9:

> Assign teachers who are best qualified in nonmathematics fields to teach classes in their specialities rather than classes in mathematics.

Problem Situation 10.—Inadequacy of funds to pay for inservice mathematics education.

Suggested Solution.—The group's overall suggestion for meeting the 10th problem situation was the following:

> Provide yearly budget allotments for continuous inservice education programs. (Seek funds from various sources if the school itself cannot provide them.)

Problem Situation 11.—Failure to recognize that inservice education should be continuous.

Suggested Solution.—The general solution emerging from the group's discussion was the following:

> Schedule inservice education activities as a regular part of the school program.

Problem Situation 12.—The inadequacy of the mathematics program to challenge realistically all levels of students' abilities.

Suggested Solutions.—The following two suggested solutions to Problem Situation 12 came out of the group's discussions:

1. Study ways and means of adapting teaching and administrative procedures, instruction materials, etc., to the abilities and maturity levels of individual students.
2. Encourage talented students to attend regular classes in advanced mathematics at the local college.

Problem Situation 13.—A shortage of well-prepared high school mathematics teachers to teach the evolving new subject matter and a shortage of college teachers to give the necessary inservice education courses.

Suggested Solutions.—Three suggested solutions of the discussion group for the 13th problem situation are the following:

1. Employ retired military personnel who have the necessary competencies in mathematics and who have taken training to teach.
2. Employ mothers whose children have grown up. Train these women while they teach, increasing their salaries as they acquire experience and additional training.
3. Explore the use of teaching machines, tape-recorded lessons, television films, and other similar devices.

Promising Practices in Medium-Size School Systems

Chairmen for the session:
DR. JOHN WAGNER
DR. BRUCE MESERVE

HIGHLIGHTS OF THE REPORT

1. Local school systems should take leadership in providing for inservice education teachers. Present practices reflect a transitional stage between initial orientation and intensive study. Emphasis should shift away from orientation to intensive study of specific courses and their underlying mathematical concepts.
2. The local superintendent should take responsibility for the continuance of the inservice program as well as for its initiation.
3. An essential provision for successful inservice of teachers is employment of a qualified coordinator.
4. General plans for inservice improvement should be presented to all teachers of a school system at a meeting.
5. Seminars on subject matter directly related to the mathematics to be taught in school classes are valuable for inservice improvement of teachers.
6. Teachers should be given the opportunity to observe a master teacher at work in the classroom as he presents the new material.
7. Teachers should be guided in reading about content changes and improved methods of teaching.
8. Local industries may be solicited for assistance in providing facilities for inservice improvement of teachers.
9. Teachers' salaries should be increased as a means of keeping the teachers in the profession and of encouraging them to undertake self-improvement.
10. As a means of becoming informed about the applications of mathematics, teachers should be encouraged to spend summers working in industries and research laboratories.
11. National Science Foundation institutes and similar programs effectively supplement local programs for inservice improvement.
12. College credit courses given to local groups of teachers can effectively contribute to inservice improvement.
13. A traveling consultant from the mathematics staff in the State department of education can assist in inservice improvement.
14. Correspondence study courses are a means of inservice improvement of teachers.

15. The following or similar topics could make up a program of 90-minute lectures providing some orientation and some intensive inservice education for mathematics teachers:

A new look at the mathematics of grades 10, 11, 12.
Mathematical content for grades 7 and 8, including number bases, systems of numeration, modular arithmetic, and programs for average and below-average students.
Arithmetic and its generalizations.
Geometry and statistics for grades 7, 8, 9.
Sets—basic definitions.
Sets in elementary algebra.
Deductions and structure of a first course in geometry.
The use of coordinates in a first course in geometry.
An introduction to abstract algebra.
Mathematics for grade 12.

16. Teachers who have participated in summer or academic-year institutes should be given an opportunity to put into practice what they have learned. These experiences will help the teachers to grow and will also benefit other teachers in the [illegible] system.

17. Lecture programs financed by the National Science Foundation and organized by the Mathematical Association of America are an effective means of inservice improvement of teachers.

THE REPORT

Our school systems, as probably never before in their history, are facing the immense task of finding and retaining qualified mathematics teachers. They are beset with the rapidly expanding birth rate of the 20th century, with the pressing demand of government, business, and society to teach more mathematics, and with the rapid

As one means of attacking the problem of personnel, local school systems need to look to teacher inservice education. Present practices in inservice education reflect a transitional stage between initial orientation and intensive study. A few years ago it was necessary to help teachers recognize: (*a*) The existence of recommendations such as those of the College Examination Board's Commission on Mathematics, and (*b*) the increasing development of experimental programs such as the program of the University of Illinois Committee on School Mathematics, the University of Maryland program for grades 7 and 8, and the program of the School Mathematics Study Group.

Many communities have passed through this era of orientation in which it is of primary importance to help teachers understand the point of view of contemporary recommendations and proposed content materials. Orientation is an important first step in any inservice education program, but the primary emphasis should shift as rapidly as possible to intensive study of specific courses and their underlying mathematical concepts.

Not only does the inadequately prepared mathematics teacher require inservice education; but also the experienced teacher, who has not attended a college mathematics class for a number of years and is therefore only vaguely aware of modern mathematical developments, also requires inservice education.

The principal responsibility for providing an inservice program rests with the local school system. Each local system has its own unique problems, which it is more interested in solving than are any other institutions or organizations. The local board of education must therefore take the leadership in setting up and carrying through an adequate inservice program for its mathematics teachers. The superintendent, as the board's professional executive, needs to make his conviction and enthusiasm for such a program clearly evident to the board members, who will be asked to approve the policy framework to the program; and also to the administrators, supervisors, and teachers, who will be directed to carry out the plans. These latter must understand, first, why the program is needed and, second, what is each person's responsibilty for its successful execution.

The leadership of the board of education and the superintendent, although absolutely necessary to initiating the program, becomes even more crucial in the continuing operation of the program. The principals, supervisors, and mathematics teachers will react immediately to any diminishing interest on the part of the board and superintendent, and the program will soon become without substance. Discouragement and frustration will be the teachers' lot.

Obviously, inservice teacher education can be carried out in different ways. Among those ways are the following three:

1. The local system may inaugurate a program which does not require cooperation from any State or National organization.
2. The program may depend upon cooperation with the State department of education of a State or local institution of higher education.
3. The program may depend on or use the cooperation of national groups and organizations.

Locally Inaugurated Programs

The inservice education of mathematics teachers should begin at home, that is, in the local school system or region, without necessarily seeking participation from National or State agencies or from colleges or universities. No one plan can meet the special needs of every local group of mathematics teachers. Local groups probably differ as much in their readiness for improvement in mathematics education as do the children in a heterogeneous class of 7th-grade mathematics.

On the assumption that the local community, its board of education, the school administrators, and the teachers of mathematics themselves feel and understand the need for inservice education, certain steps can profitably be taken.

The first of these is the employment of a mathematics coordinator who will spark teacher interest and participation in an improvement program and follow through continuously. The coordinator must have vision as well as superior preparation for his task. He must know many things about children and learning, teachers and teaching, administrators and administration. High on the list of such competencies must be the depth and breadth of his command of mathematics itself.

It is desirable that the preparation of the coordinator in mathematics should include becoming familiar with the mathematical concepts necessary for implementing programs of such nationally known groups as the Commission on Mathematics of the College Entrance Examination Board, the School Mathematics Study Group, the University of Illinois Committee on School Mathematics, and the University of Maryland Mathematics Program.

There are several organizational aids which the coordinator can use to further the inservice education of the teachers.

One organizational aid is to hold meetings of the mathematics teachers, whether or not they are organized in departmental or interdepartmental groups. At these meetings there might be presentations of plans for improving or modernizing curriculums, lectures on mathematical topics, and group discussions (with or without panels); of curriculum proposals, mathematical concepts, and teaching procedures. Films showing proper presentation of new mathematical topics or teaching demonstrations might be used. At most of these meetings the coordinator should provide the leadership, but he should also utilize teachers' contributions.

A second promising procedure is the use of seminars conducted by a subject-matter specialist who deals with content related to a specific secondary school course. The advantage of concentrating on one course instead of several is that there will be less chance of teachers coming to know only "a little bit about a lot of things."

The seminars should not be conducted by means of lectures alone. Experience with institutes indicates that teachers appreciate such teaching procedures as: Gaging the pace of instruction by periodically directing questions to the group and requesting questions from the teachers; adequately reviewing the background needed for the new work; preceding a rigorous presentation with an intuitive, motivated approach; using many illustrations from school mathematics and daily life; using visual presentation via the blackboard;

assigning problems and dealing with them at the next meeting of the group; and giving references for supplementary reading and study.

A third practice that has much value is to provide released time for teachers during the school day to observe a master teacher demonstrate with school students the teaching of experimental course lessons. If the master teacher can follow the demonstration by discussing with the observers the rationale of his teaching acts and can answer questions, the benefits to the observers will be increased.

Another type of experience that will improve the teachers' inservice preparation is the provision, at frequent and regular intervals, of released time during the schoolday for group meetings devoted to the study and development of the mathematics curriculum.

Group meetings are important in these times, when it is not uncommon for changes to be made in one part of the curriculum without reference to what precedes and what follows. A curriculum that has yawning gaps and excessive overlapping is a poor curriculum.

Finally, having mathematics teachers at different grade and subject levels look at the curriculum as a whole is more likely to develop group solidarity and an *esprit de corps* than sole reliance on sporadic, individual experimentation.

Membership on a textbook selection committee is one method of providing a teacher with a knowledge of the new content and ideas in mathematics. In order to judge the relative merits of a number of competing textbooks, the teacher must become acquainted with the mathematics contained in each of them. Since an increasing number of books are being produced, frequently embodying many of the proposals of the various study groups working in mathematics, the teacher often must do some independent study if he is to make an intelligent appraisal. The natural consequence of this is to broaden the teacher's knowledge and understanding of some of the new trends in mathematics.

Many teachers unprepared to teach the new concepts of mathematics which are beginning to be taught at the secondary level are handicapped because they do not know what to read and study in order to correct their deficiencies; or, if they know, they do not have readily at hand the requisite source and reference materials. Hence, it is highly desirable for each school system to provide guides for individual teacher reading in mathematics. To be of maximum benefit, such a guide should include specific references and should list the topics recommended for study. If this program is to be really effective, each school system should provide and have this reference study material readily available.

Federal and State support for improvement of science and mathematics education has been growing and is very helpful, but, as already emphasized, these efforts alone cannot do the job. More basic to the success of the overall program is local support, and one very effective method is to interest local industries in supplying expert assistance or financial aid—or even both, if possible. Support from local industries would undoubtedly be a powerful stimulus for local teachers to participate actively in the program.

The low salaries of teachers compared to the salaries of other professional persons have been well publicized. In mathematics it is well known that many competent teachers have found it necessary or desirable to change to better paying occupations. One of the ways to increase the salaries of teachers and thereby to help keep them in the profession is to employ them on a contract covering more than the customary months of service. This additional time could be used advantageously for curriculum development and inservice education. The increased knowledge and proficiency gained by the teachers and the resulting benefit to the children would amply justify the additional expense.

A common complaint about the teaching of mathematics is that it is based solely on the knowledge that the teacher has gained from books and from formal education. Too often teachers are unfamiliar with the applications of mathematics to science and industry. An excellent remedy for this lack is for local industries which use mathematics in their engineering, production, and research processes to employ mathematics teachers during the summer months. Not only will the teachers get new insights into mathematics and how it can be applied, but the experience will likely provide the stimulus for further study as the need becomes evident. Furthermore, the teachers will be better able to motivate the pupils as a result of being able to speak from direct experience rather than from hearsay.

Cooperating Programs

The necessity for national upgrading in any specialized subject like mathematics is usually a reflection of the deficiencies existing at State levels. The responsibility for correcting the deficiencies is therefore primarily the task of the State department of education and local school districts.

In fulfilling this obligation, the State utilizes its own special offices in the particular field, the local resources of trained personnel in the subject, and the existing professional centers of education whose work will insure implementation of the program in the future. National assistance in the form of Federal funds, materials, personnel training, etc. is intended to supplement and not to substitute for individual State initiative. The following cooperative activities on the State level can reasonably be expected as the first steps for improving the mathematics curriculum.

College-Credit Courses

The State and private institutions of higher education, with competent professional mathematicians, are in a position to train teachers in various mathematical fields. Special courses must be carefully planned for those teachers whose background education did not include the elements of today's mathematics. For those teachers who have recently graduated and who possess some familiarity with the new mathematical concepts, advanced courses should be developed to supplement their college work.

College credit should be given for the mathematics courses for teacher inservice education. However, since many of these courses will not be listed as regular offerings for the usual M.A. and M.S. degrees, the credits can be applied to degrees such as the "master of arts" or the "master of sicence in the teaching of mathematics" (M.A.T., M.S.T.). In special cases, where individual colleges do not give credit, the State department of education could recognize the courses as fulfilling some of the certification requirements.

Instruction could be held on the college or university campus. In some cases it might be preferable (because of the distance or time element involved) to have the instruction in one of the centrally located buildings of the school system. In 1958 and 1959 Montclair State College (New Jersey) provided ten 90-minute lectures for teachers of the schools belonging to the Metropolitan School Study Council. These were curriculum-survey sessions. The program is briefly described below in order to suggest some of the specific topics that can be covered in such sessions (undoubtedly there are many other topics of equal value):

1. Orientation, a new look at grades 10, 11, and 12.
2. Mathematical content for grades 7 and 8 including number bases, systems of numeration, modular arithmetic, and programs for average and below-average students.
3. Arithmetic and its generalizations.
4. Geometry and statistics for grades 7, 8, and 9.
5. Sets—basic definitions.
6. Sets in elementary algebra.
7. Deductions and the structure of a first course in geometry.
8. Coordinates in a first course in geometry.
9. An introduction to abstract algebra.
10. Mathematics for grade 12 (preparation for calculus).

The above-mentioned orientation program was accompanied by an inservice study program that consisted of 6 full days during which the teachers worked on materials for classroom use. The sessions were held on schooldays spaced at intervals of about 2 weeks. The teachers were released from their classes for this study program.

Certificate-Renewal Courses

Problems of teacher certification are frequently local, varying greatly from region to region. One immediate use that can be made of the special credit courses in mathematics is for certificate renewal. An attempt should be made to require credit courses for certificate renewal in the particular subject matter which forms the major portion of the teacher's class instruction. For teachers of mathematics, the special mathematics courses could carry credit for certificate renewal in those cases where the credits are not directed toward an academic degree.

Traveling Consultant

Unification of a school system's mathematics program might be achieved by a mathematics consultant traveling under the auspices of the State department of education. Such a consultant should be a trained mathematician (preferably of professorial rank) rather than a member of the administrative staff of the State department of education. The specific task of the consultant would be to visit regularly the various schools of a system. He would give lectures on special topic in contemporary mathematics, advise teachers in the selection of text material, suggest procedures or methods of teaching, keep teachers informed of contemporary trends and new developments in mathematics, etc.

In regions where no provision can be made for teachers to attend special credit courses in mathematics at a college or university, the traveling consultant is perhaps the next best solution to the problem of teacher inservice education in mathematics.

Television Courses

For States where college credit courses would not be available and where a traveling consultant's activities would be limited, an effective inservice training program could be organized on television. Written material for teacher study should form an intergral part of such a program. An examination would be given at the end of each lecture series. Credits for successful completion of a series could be applied toward certification renewal.

Where television facilities are not readily available, correspondence courses might be offered, with units developed under the auspices of the State department of education or the local college. Each course would consist of units to be completed by the teacher, corrected by the college staff, and returned to the teacher. An examination would be given at the end of each course. Credit could be applied toward one of the special college degrees in teacher inservice education or accepted by the State for teacher certification renewal.

National-Level Cooperation

Many activities in the areas of curricular revision, course content development, and teacher inservice education at the national level are receiving substantial monetary support from various sources, including the Federal Government, private industry, and educational foundations. A school system can derive maximum benefit from these activities by giving strong encouragement to its mathematics teachers and supervisors to participate. This encouragement can take several forms: Encouraging individual teachers and supervisors to become acquainted with these national efforts and encouraging them to take an active part in the activities, furnishing financial aid to the participating teachers to supplement any support offered by the programs, and, most important, recognizing the teachers' efforts afterward by giving them full opportunity to make use of the experiences in their own teaching and in working with their colleagues. Specific instances of ways in which maximum use can be made of the national programs are suggested below.

A school system can select some of its promising young teachers, as well as its key teachers and mathematics supervisors, and encourage them to apply for participation in the seminars and workshops conducted by such groups as the School Mathematics Study Group, the University of Illinois Committee on School Mathematics, and the University of Maryland Mathematics Program.

The school system can relieve these teachers from some portion of their teaching duties, or especially their extracurricular duties, to enable them to participate in these national efforts as members of writing teams or as participants in conferences and workshops. But for the dedicated teacher, it is most important that he be allowed to make use of the experiences thus obtained in his classroom work and in his school program generally.

Similarly, teachers who have participated in National Science Foundation Institutes—summer, inservice, or academic year—should be given every encouragement to build upon this background. Those teachers who have not had recent refresher work, or those who desire advanced training, should be encouraged to attend. Encouragement in the form of college credit and travel allowance, is often given to teachers to attend university inservice programs which meet late afternoons or on Saturdays. The school system can give further strong encouragement through adjusted duty schedules so that the teachers will have sufficient time for study and lesson preparation.

Many secondary school mathematics teachers and some supervisors are enabled to spend a full sabbatical year in advanced mathematics study through the Academic Year Institutes of the National Science Foundation. About 700 are currently participating, many of them at a great personal sacrifice because of the cost of moving their families and the fact that stipends do not match their regular salaries or their combined family incomes. The local board of education can provide material assistance in the form of supplementary salary grants. Frequently this is possible by awarding the teacher on leave the difference between his salary and that of a substitute. The board also can help the teacher by making provision for his continued membership in the pension fund during his absence. Through these benefits and through a liberal policy of granting leave of absence without loss of status the board can assure that the teacher will want to remain in the local system.

School systems should take advantage of the lecture programs sponsored by the National Science Foundation and conducted by the Mathematical Association of America and by the various State academies of science. These provide mathematicians who will lecture to students or meet with teachers to discuss mathematics and mathematics education.

The American Association for the Advancement of Science, under a grant from the National Science Foundation, maintains a traveling library of mathematics books for high school use. Teachers and librarians should become familiar with this collection. A list of the books, together with similar lists of books suitable for elementary school use, is available from the American Association for the Advancement of Science.

A local school system can organize small teacher study groups which will use the Commission on Mathematics' materials or other similar materials in connection with their own curricular efforts.

Teachers and supervisors should be encouraged to belong to the professional organizations in the field of mathematics teaching, such as the National Council of Teachers of Mathematics and the Mathematical Association of America. Opportunities should be made available for them to attend meetings of the NCTM and meetings of its affiliated State councils, or the sectional meetings of the MAA, which are increasingly being devoted to topics of interest to high school teachers.

"The Continental Classroom", which has presented to a nationwide TV audience early morning classes in physics and chemistry, has provided similar classes in mathematics with emphasis on teaching the subject. Teachers should be encouraged to participate in any related educational programs and given appropriate recognition for completion of a course.

The ultimate importance of these national efforts is, of course, in their eventual application at the local level. By enabling their teachers and supervisors to make optimum use of the available opportunities under the best possible conditions and by following up with opportunities and freedom for the participants to make use of the knowledge gained, local school systems can reap maximum benefits.

Suggested Patterns

Now is an opportune time to initiate forms of action which will take advantage of recent proposals for renovating the mathematics curriculum. It is possible to base inservice education programs on the intensive study of instruction materials which will ultimately be presented in the classroom. One fruitful method is comprised of the following steps:

1. Select a single course for intensive study, such as ninth-grade algebra.
2. Select instruction materials for this course as prepared by the University of Illinois Committee on School Mathematics, the Ball State Teachers College Program, etc. The entire mathematics staff should then study the materials intensively, for one reason because the materials contain commentaries for teachers to use in explaining and amplifying the students' textbook.
3. Devote at least one semester, and preferably 1 year, to study of the materials selected. The study sessions should be of approximately 2 hours' duration, at least once a week, and at a time convenient for all members of the mathematics staff. The staff should regard these sessions as an opportunity to understand the new materials in relation to the total mathematics program: Discussing the student textbook, topic by topic; solving the problems; constructing tentative tests and examinations; and carefully studying the related teachers' commentary.
4. Have a qualified consultant work with the mathematics staff during the preliminary study sessions and throughout the first year of classroom presentation of the materials.

Junior high school teachers may apply the same procedures to a study of recent materials for grades 7 and 8. Senior high school teachers may also discover that these materials have important connotations for pupils who are now assigned to the general mathematics sequence.

Another effective course of action pertains to the need for more advanced mathematics in high school for mathematically gifted pupils. For example, the advanced placement program requires that a course in the calculus be provided for gifted pupils in grade 12.

Generally speaking, there are two conditions for the effective presentation of such a course:

1. An effective sequence of courses leading to the calculus should be established in grades 7 through 11.
2. The teacher should be adequately prepared for teaching such a course. Local universities and colleges should provide appropriately designed refresher courses in the calculus and related areas for those teachers who are to present the subject in grade 12.

Films may be used as an aid in inservice education. They can bring to a group of teachers expositions of subjects carefully prepared by teachers who have special interest both in the field and in teaching. However, films are no substitute for college consultants.

Many desirable and very different initial orientation programs are now in effect; many desirable and very different intensive inservice education programs are also possible. The following questions and answers may be helpful in appraising the relative merits of different types of programs for particular schools and situations.

1. *What sources can help the need for teacher inservice education to be felt more keenly?*
    a. Local agencies.
    b. Outside agencies.
2. *What means are available for initiating the inservice education?*
    a. Consultants.
    b. Finances.
3. *How can the following types of organization contribute to an inservice education program?*
    a. Local or district seminars with internal leadership.
    b. Seminars with occasional or regular participation by consulting mathematicians.
    c. Formal university extension classes.
    d. Television or film instruction.
    e. Home study courses.
4. *What kind of materials are most relevant?*
    a. Experimental classroom materials (e.g., School Mathematics Study Group, etc.).
    b. Reference books such as the yearbooks of the National Council of Teachers of Mathematics.
    c. Films and filmstrips.
5. *What is a reasonable time schedule for intensive inservice education?*
6. *What things can facilitate the teachers' individual study?*
    a. Library materials.
    b. Teaching machines.
    c. Study guides.
    d. Films.

Promising Practices in Large School Systems and State Departments of Education

Chairmen for the Session:
MISS VERYL SCHULT
DR. EUGENE P. SMITH

HIGHLIGHTS OF THE REPORT

1. Learning the new mathematics is a necessary part of a teacher's activities.

2. A good inservice education program serves the needs of *all* persons concerned with instruction: Teachers, supervisors, administrators.

3. A good inservice program is continuous.

4. Large school systems have many opportunties to provide effective inservice education by means of conferences, panel discussions, reports, workshops, library facilities, and institutes.

5. Formal and informal study groups provide inservice education of high order.

6. Independent reading and study are excellent means for achieving individual teacher improvements.

7. Inservice education programs need to be evaluated carefully.

8. Curriculum evaluations and revisions provide good inservice improvement experiences.

9. Experimentation with new materials and methods provides opportunities for inservice growth.

10. Demonstration lessons followed by discussions contribute to inservice improvements.

11. Professional teacher organizations exist mainly to provide opportunities for inservice educational improvements.

12. Each State department of education should assume active leadership in inservice improvement of teachers.

13. Each State department of education should have on its staff a consultant in mathematics who can provide specialized help for inservice education programs.

14. New York State has taken the lead in providing television and kinescopic films.

INTRODUCTION TO THE REPORT

Continous learning of new mathematics by secondary school mathematics teachers is an essential part of their professional activities. Today, this is more important than ever before, due to the vast changes which have taken place in both the content and the spirit of mathematics during the past few decades.

The many ways by which the mathematics teacher can stay abreast of the new content and methods include group study as well as individual efforts in learning and transferring content and method into classroom practice.

The basic purposes of inservice programs for teachers are achieved only if the end result is that students learn more mathematics. That is, the success of the programs is measured by the benefits the students derive from improved mathematics instruction.

It is possible to identify a number of general characteristics of a good inservice program that contribute to its success. It is necessary that all the features of any one program should revolve about a central purpose; that is, to provide such new insights into mathematical concepts that the teachers will be able to translate the new knowledge into classroom practice.

When enumerating and discussing ways in which teachers may keep abreast of mathematical developments pertinent to mathematics teaching, one must recognize that the teachers must be given financial support and released time. The sources of financial support are generally centered in private and government foundations, State departments, county systems, and local systems. The local administrators should recognize their obligations to provide teachers with released time from professionally irrelevant tasks in order to keep up in vital professional activities.

THE REPORT

Desirable Organization Patterns

No single ideal or best organization pattern for teacher inservice education exists, of course. Some possible patterns, however, are suggested below:

1. **Courses or seminars offered for credit by—**
    - a. **College and university instructors in person.**
    - b. **College and university instructors via television. (Outside foundations, universities, State departments, or strong local efforts may finance the courses.)**

2. Noncredit courses or seminars offered by university instructors and financed by—
   - a. City, county, or State.
   - b. National Defense Education Act.
   - c. Foundations (National Science Foundation, Carnegie, Ford, etc.).
   - d. Organizations (Association for Computing Machinery, etc.).
3. Academic year fellowships offered by National Science Foundation.
4. Fellowships for summer study offered by—
   - a. National Science Foundation.
   - b. General Electric.
   - c. Esso.
   - d. Shell Foundation.
   - e. Others.
5. Departmental meetings, with lectures, panel discussions, or reports on such subjects as—
   - a. The work of the School Mathematics Study Group or of the Commission on Mathematics (College Entrance Examination Board).
   - b. The University of Maryland Mathematics Project (UMMaP—Jr. H.S.).
   - c. How the new mathematics is affecting standardized testing.
   - d. Special projects.
   - e. Yearbooks and other publications (National Council of Teachers of Mathematics and others).
   - f. "Modern" mathematics.
   - g. Gifted students of mathematics.
   - h. Correlation of mathematics and science.
   - i. Uses of mathematics in industries.
6. One- or two-day conferences utilizing—
   - a. Talks by outside speakers, consultants, or visiting lecturers.
   - b. Discussion groups to correlate the work of elementary and secondary schools.
   - c. Cooperative discussions with representatives of high schools and colleges.
   - d. Reports and discussions of curriculum experiments.
   - e. Study groups led by—
     - teachers who have served on curriculum committees.
     - teachers who have special interests or have made special investigations.
     - teachers who have had recent training in institutes.
     - supervisors.
   - f. Reviews and discussions of new mathematics books.
   - g. Discussion groups on pertinent problems in mathematics education.
   - h. Community resources.
   - i. Study and discussion of the development of mathematical concepts.
   - j. Use of learning aids.
7. Informal study groups working together on specific problems of mathematical instruction, such as—
   - a. The place of proof in junior high school.
   - b. The place of proof in algebra.

- c. The extent of rigor in geometry.
- d. The place of solid geometry in the high school curriculum.
- e. The place of standardized tests in evaluating instruction.

8. Workshops (1 or more weeks) devoted to such activities as—
   - a. Studying the new instructional materials.
   - b. Viewing and evaluating mathematics films.
   - c. Viewing demonstration teaching with followup discussion.
   - d. Studying particular areas of mathematics.
   - e. Studying new mathematics tests.
   - f. Developing materials for special groups of students (gifted, remedial).
   - g. Investigating practices in other school systems.

9. Work on committees concerned with such projects as—
   - a. Selecting standardized tests.
   - b. Evaluating and testing in the mathematics department (preparing tests, helping in correcting, interpreting results).
   - c. Setting up criteria for selection of textbooks.
   - d. Selecting textbooks.
   - e. Selecting learning aids, such as films, models, instruments.
   - f. Curriculum study and production.
   - g. Science and mathematics fairs.
   - h. Mathematics contests and other competitions.
   - i. Planning programs for mathematics clubs.
   - j. Planning mathematics classrooms in new schools.
   - k. Mathematics presentations on educational television.
   - l. Preparing bibliographies for use by other teachers.
   - m. Collecting and/or preparing career guidance material in mathematics.
   - n. Planning work for special groups of students (talented, slow, etc.).
   - o. Preparing study guides for teachers, such as bibliographies, glossaries of the vocabulary and symbolism of the new mathematics.

10. Trips to places of mathematical interest such as "digital computer centers."

11. Participation in the work of mathematical organizations (Mathematical Association of America, National Council of Teachers of Mathematics).

Inservice Programs: Some Guiding Principles

1. The central theme of a program should be "mathematics and its teaching."

2. Mathematics teachers should play key roles in structuring their inservice education program.

3. Genuine support by administrators is essential: It should grow out of their interest in supporting teachers' efforts.

4. Careful and enlightened planning of activities is essential.

5. Plans for a program should take into account the competencies and individual differences of the teachers for whom the program is intended.
6. A climate conducive to a free interchange of ideas and viewpoints is desirable.

7. Evaluation programs should be carefully planned and conducted. Results should be used in planning other programs.
8. A variety of texts and professional books and instructional materials should be easily accessible to teachers.
9. Teachers' improved competence through participation in inservice programs should be reflected in promotions, in pay, and in other forms of recognition.
10. Intrinsic and extrinsic incentives for teachers' participation should be provided.
11. All posible educational and industrial resources should be used to the fullest extent.

Checklist for Judging a Workshop

Question	*Yes*	*No*
1. Is the workshop related to the needs and problems of the teachers?		
2. Has there been adequate preplanning?		
3. Is the workshop conducted under competent and informed leadership?		
4. Are the necessary materials and facilities made available?		
5. Do the teachers feel they can use in their own classrooms the background and skills developed in the workshop?		
6. Do the teachers have an opportunity to report practices and outcomes to other staff members?		
7. Are administrators invited to participate?		
8. Is attention given to self-evaluation by the participants?		

Role of the Large School System

The large school system offers unique opportunities for inservice teacher education. Its limited geographic area facilitates ready communication and affords homogeneous conditions. The industries of a city interact with the educational system to give teachers both stimulation and service. The city or area colleges make formal courses possible and furnish an atmosphere conducive to continuous study. Large schools can permit greater flexibility among their faculty, and thus allow released time for inservice education. The large community population makes it easier for schools to find qualified substitutes for this purpose. The big student bodies make it possible to have not only homogeneous grouping and a greater variety of courses but also better formal and informal research on curriculum, methods, and organization. Since great numbers of teachers are employed it is feasible and efficient to appoint department heads and supervisors to assist with inservice education. Library facilities are usually good in cities and large school systems, and this fact enables teachers to study individually or in groups more effectively.

More specific ways in which the advantages of large school systems may be utilized in inservice education are described in the sections following.

Elements of a Good Inservice Program for a Large School System

Basically a good inservice training program should be functional and should be the product of a carefully coordinated plan of organization and administration. In a large city system every advantage should be taken of factors such as size of teaching staff, supervisory staff, curriculum, and guidance personnel.

A program designed especially for mathematics teachers in the secondary schools will take cognizance of the manpower needs, problems of staffing, developments and trends in the new mathematics, and the status of mathematics teachers in the community. Special attention should be given to the problems of articulation between the secondary schools and the colleges. Hence, college admission requirements and the advanced placement program also deserve consideration.

Although there can be no one best plan to fit the needs of every local situation, a good inservice program may be described as one that:

1. Encompasses the needs of *all persons* engaged in the teaching of mathematics in the schools, i.e., teachers, supervisors, and administrators.
2. Is organized on an all-school, or systemwide, basis.
3. Is coordinated within school levels (elementary, junior high and high school) and from one grade to the next, through grade 12.
4. Is developmental in that it is continuous and adjustable to the needs of the classroom teacher and the changes in curriculum development and implementation.
5. Recognizes the individual differences among teachers, differences that reflect preservice training, length and variety of teacher experience, and grasp of subject matter.
6. Stimulates wholesome teacher growth and understanding of various viewpoints and competencies.
7. Makes provision for teachers to play some part in setting up the program of inservice training (by surveys, suggestions, and expression of preferences).
8. Is developed in an emotional climate which inspires confidence in the participants and indicates administrative support.
9. Takes into account the needs and abilities of *all* the children; the slow learner, the retarded, the underachiever, the average, and the gifted.
10. Requires the leadership of a competent and fairminded administrator.
11. Is closely integrated with the supervisory and guidance program of the school.

12. Develops a variety of approaches to teacher growth (workshops, seminars, institutes, study groups, committee work).
13. Is judged for its effectiveness at regular intervals through various techniques of appraisal.
14. Draws upon the resources of the community, the colleges and universities, and the professional staff itself.
15. Is aware of the limitations on teacher time and daily load, and makes reasonable adjustments for teachers participating in the program.
16. Gives recognition and encouragement to active participants in the program in terms of promotional opportunities, financial remuneration, salary increases, and professional status.

Practices on City and County Levels

Inservice education programs on city and county levels are becoming more numerous. Beginning below, a few of these programs are described in varying degrees of detail. There is no attempt to make this section exhaustive.

The 1959-60 Miami (Fla.) Program

The Miami program was in the form of an institute supported by the National Science Foundation. The funds provided by the Foundation covered the cost of tuition and transportation for participating teachers.

The program consisted essentially of three phases: (*a*) courses and seminars, (*b*) instructors' visits to high schools, and (*c*) special lectures.

The courses, offered by the University of Miami, were as follows: "Basic Concepts of Modern Mathematics"—Symbolic Logic, Introduction to the Theory of Groups (3 credits); "Introduction to Modern Algebra" (4 credits); "Number Systems and Elementary Functions"—Elementary Theory of Groups, Rings, Integral Domains, Fields, and Isomorphisms (4 credits).

The seminar was on curriculum (one credit). Since a part of the seminar was concerned with the problems of teaching mathematics the instructor paid visits to several high schools in the city of Miami.

The special lectures were the following: "The Contributions of Geometry to Mathematics" (by R. L. Wilder), "The School of Mathematics Study Group" (by E. G. Begle), and "Finite Projection Planes" (by A. A. Albert).

In addition to institute participants, all mathematics teachers in Dade County were invited to the lectures.

The Baltimore City (Md.) Program

The 1959 Program of Inservice Education for the Baltimore City Schools is an example of what can be achieved through workshops arranged for both teacher and pupil participation.

A 6-week summer workshop was held on modern mathematics. There were demonstration classes at both junior and senior high school levels. Teachers observed these classes and then met together each day for discussions of their observations, and for further study of topics from modern mathematics. The two student groups were thirty 8th-grade and 9th-grade students and thirty 10th- and 11th-grade students. Thirty mathematics teachers participated: 15 junior high school and 15 senior high school.

The program for junior high school students was very similar in nature to the University of Maryland Mathematics Project units for grades 7 and 8.

The content for senior high school included elementary set theory, structure of algebra, inequalities, number systems, and the nature of proof in algebra.

The 6-week session proved beneficial to both students and teachers. Some of the teachers who attended are currently conducting experimental classes and others among them are enriching their regular classes by introducing some of the modern methods and the new content when it seems appropriate. The experience provided a springboard for some of the participating students to do further reading and projects related to some of the new content introduced during the summer session. Another significant sidelight is that "old guard" teachers have seen a new interest in their students, and hence have become more receptive to some of the new content and have enrolled in an academic-year workshop similar to the summer workshop described above.

In the academic-year workshop teachers met twice monthly from October to May, at the various junior and senior high schools. The workshop was designated as "A Content and Demonstration Class in Modern Mathematics." Teachers participated by giving reports on such topics as elementary group theory and the introduction to Boolean algebra.

In Baltimore County, groups of mathematics teachers have participated in developing a program and in identifying content areas where inservice instruction is needed. For example, one group set up an advanced placement program for superior and gifted students. The mathematics supervisor has provided inservice instruction by speaking to teacher-student groups on "Simple Artin Braids as a Noncommutative Group" and "The Real Number System." Several mathematics teachers, at their own request, met with the supervisor once a week to review the calculus and to develop meaningful approaches to teaching this subject to high school students.

A mathematics department library for teachers and students has been established through the support of National Defense Education Act funds. Large senior high schools received a library allotment of $250 and small senior high schools, $100. Regular library funds are also being utilized for mathematics books for both teacher and student use.

Individual professional development of teachers is being promoted by the availability of resource materials, by direct encouragement, and by the supervisor's visits and conferences with individual teachers.

Three staff teachers were employed for 6 weeks during the summer of 1959 to set up a new geometry course for grade 10, fusing plane, solid, and elementary coordinate geometry. The committee recommended the revision of present geometry courses for *all* students to include both two-dimensional and three-dimensional concepts. Subsequent reports by Baltimore teachers indicate enthusiastic acceptance of the new 10th-grade course.

Cooperation among Baltimore County, Baltimore City, and Johns Hopkins University is reflected in the Esso Foundation Inservice Institute's Foundations of Mathematical Science, the National Science Foundation's summer institute in Secondary School Mathematics, and in the development of special courses on Modern Mathematics for the Secondary School, and Introduction to Probability and Statistical Inference.

The Seattle (Wash.) Program

Seattle has attempted to resolve the inservice education problem through a continuous offering of inservice courses at different levels. The high school mathematics courses are the following:

For junior high school teachers, a modern approach to teaching arithmetic and a study of the University of Maryland Project and School Mathematics Study Group materials.

For junior and senior high school teachers, the appendices of the report of the Commission on Mathematics of the College Entrance Examination Board.

A television course on analytic geometry and the calculus given by a teacher in half-hour lessons 4 days a week affords the regular teachers opportunity to watch the television demonstration for the first part of the lesson. The fifth day, and for the remainder of the other four class periods, the regular teacher is responsible for the instruction.

The School Mathematics Study Group algebra text is being studied by over 100 teachers.

The school system reports that outstanding mathematics teachers from its own staff have proved to be the best instructors for the inservice classes.

The Newton (Mass.) Program

The administration of the Newton Public Schools feels a responsibility for helping teachers financially to get the continuing mathematics education necessary to keep up to date mathematically.

During the summer of 1959, some of the elementary school teachers, with the aid of an arithmetic consultant from Tufts University, wrote four units to be used in the fourth, fifth, and sixth grades in 10 of the city's 26 elementary schools. These units were enrichment material and the consultant worked with the teachers who taught the material in 1959–60. The next step will be to get mathematics teachers in all the 26 elementary schools trained to use this material. The final step will be to provide for vertical development in the elementary mathematics curriculum.

For junior and senior high school teachers the head of the mathematics department is teaching an inservice education course that meets each Wednesday afternoon from 3:20 to 5 p.m. and carries one inservice credit per semester. In 1959–60 the units formulated by University of Illinois Committee on School Mathematics, "Mathematical Induction" and "Exponents and Logarithms", were discussed the first semester and the geometry courses formulated by the School Mathematics Study Group, the second semester. These Wednesday afternoon sessions have been running since March 1957, and will continue using the materials developed by UICSM and SMSG.

In 1959–60 two junior high school mathematics departments studied the problems of ability grouping as well as the Maryland and SMSG seventh- and eight-grade programs.

The San Francisco (Calif.) Unified School District Program

The San Franciso district has sponsored several inservice training courses using instructors from the University of California. The district has also provided funds for a lecture series for mathematics teachers. Lectures by such men as Hermann Baravalle and George Polya, emphasizing mathematical content, enrich the backgrounds of secondary school mathematics teachers. Recognizing the values of employing teachers during the summer for special projects, the district employed six mathematics teachers during the summer of 1959 to develop enrichment materials for superior students.

The Racine (Wis.) Program

The distinctive feature of the Racine plan is its long-range sequential view. The inservice activities for the past 3 years have been centered around the following:

1. "Current Trends in Mathematics Education," designed to stimulate interest and curiosity and a feeling of need for information.
2. "The Modern Approach to Mathematics and Its Teaching." A college professor gave a course on modern mathematics and its relation to the secondary school program.
3. Curriculum revision (half time); "Pure mathematics"—University of Wisconsin extension course (half time). The curriculum revisions were made in the light of the 2 previous years' study and of current thinking. The university course was intended to enrich the teacher's background.

Another interesting project of the Racine schools is a workshop for principals. The purposes of this workshop are to—

Make the principals aware of their teachers' problems

Help the principals schedule classes according to the readiness of individual staff members to meet course requirements

Inform the principals of curriculum changes.

The Philadelphia (Pa.) Program

The Philadelphia inservice mathematics program is part of a continuing overall program of inservice education for teachers of all subjects at all levels, and represents a definite policy of the school system. The program is under the direction of the Associate Superintendent, Dr. Helen C. Bailey, who is in charge of curriculum.

Direct inservice education operates through (1) a 5-week summer workshop conducted each year since 1941; and (2) school-year courses given after school hours in central locations throughout the city.

An indirect but very significant phase of Philadelphia's inservice program is that of curriculum construction, a very carefully organized activity. Curriculum committees composed of teachers, department heads, principals, and curriculum staff members are constantly at work evaluating, improving, and rewriting the many courses of study. For example, in mathematics the curricular work (from kindergarten through grade 12) is coordinated and articulated through a Mathematics Curriculum Policy Committee. Each school level also has its curriculum policy committee in mathematics, members of which serve on the overall policy committee. In addition, there are many other working committees, one for each subject. Substitute service is provided for teachers where required, and extra remuneration is given for Saturday and summer work.

The fact that curriculum committees are representative in membership, that flexibility of instruction, experimentation and pilot runs are planned procedures, and that teacher reactions are continously sought results in curriculum construction's being an integral part of the inservice program for the improvement of instruction.

Some significant features of the Philadelphia program are the following:

1. Inservice credits are given for completing inservice courses of 14 hours each. These credits may or may not be combined with college credits to a total of 30 in order to receive the master's degree equivalent of $400 on the salary schedule.
2. Courses are taught primarily by specially qualified teachers of the Philadelphia school system. Remuneration is at the rate of $5.50 per hour. In addition, college mathematics professors have been used occasionally. Consultants, experts in their respective fields, have been secured to advise the curriculum committees.
3. Suggestions for courses and for course content are solicited from a number of key persons, including members of curricular and professional committees, principals, school officials, former course leaders, specialists in the subject; and from the teaching force at large.
4. Wide publicity is given to the program through the school district's monthly house organ "School News and Views," and through other media. Superintendents, principals, and department heads encourage their teachers to participate.
5. Mathematics teachers are encouraged to apply for (and many have attended) the summer sessions arranged by the National Science Foundation and similar agencies.
6. Guidance and concrete help for the inservice work and for curricular experimentation is secured through the School Mathematics Study Group.
7. Costs of the program are met almost entirely through the school budget, although some additional funds are available through a local foundation.
8. Teachers may avail themselves of the facilities of an outstanding professional library in the school administration building.
9. Fully equipped and well-manned departments for audiovisual aids and for television and radio are used in the teacher-improvement program.
10. The school staff includes competent supervisors and teacher aids, although no one person has responsibility for supervision of secondary mathematics.

Twelve mathematics courses given recently under the Philadelphia program are the following:

1. Mathematics for grades 7 and 8 (SMSG).
2. First Course in Algebra (SMSG-grade 9).
3. Geometry (SMSG-grade 10).
4. Intermediate Mathematics (SMSG–grade 10).
5. Calculus and Analytic Geometry.
6. Elementary Functions and Matrices.
7. Finite Mathematics.
8. Modern Mathematics.
9. Univac Programing.

10. **Modern Views of Mathematics (theory of sets, mathematical logic, groups, etc.).**
11. **Computer Concepts.**
12. **Expanding Horizons in Secondary and College Mathematics.**

The Role of State Departments of Education

The role of State education departments in inservice programs has been mostly regulatory. The history and degree of State control of subject matter varies widely, and the part played by the departments in the mathematics programs has been very different among the States. In undertaking such a program any State must necessarily begin with its own situation. This discussion assumes that the program can make use of statewide or regional meetings, pamphlets and bibliographies, consultation services, updated curriculum, and instructional aids. It is not assumed, however, that a State department will be the only agency offering inservice improvement by these means. City school systems or other school organizations within the State may likewise utilize the same means and there can well be coordination and cooperation between the administrative units.

Regional and Statewide Meetings

A number of ways are open to a State department of education to organize meetings of mathematics teachers for considering problems related to changes in course content and teaching methods. First, it can take direct action to provide chairmen, speakers, consultants, panelists, and discussion leaders; and issue invitations to teachers. Second, it can work through the State teachers association to share the responsibilities. Third, it can likewise cooperate with State, regional, or local organizations of mathematics teachers. Fourth, it can cooperate with national organizations of mathematics teachers to set up conferences, provide lecturers, and give professional leadership.

Total funds for the meetings can come from the State or from local governmental or organizational funds; or the funds can come part from one source and part from another.

Agenda for the meetings are easily arranged. A look at the titles of papers, talks, and discussions of any national meeting of mathematics teachers, or at the titles of articles in any journal of the profession—or a look at recent yearbooks of the National Council of Teachers of Mathematics, or at reports issued by the Commission on Mathematics of the College Entrance Examination Board—will suggest relevant subjects. In general, the agenda will relate to changes in content of mathematics courses and to methods of teaching the new material.

Pamphlets and Bibliographies

One service for which a State department of education has the responsibility is to publish a curriculum which sets a good pattern for selection and organization of subject matter. Such a publication is in itself a strong influence in inservice improvement of teachers, but it exerts a much stronger influence through training those teachers who are called on to help prepare the curriculum. The discussions in committee constitute a valuable phase of reeducation of participants and through them of many other teachers in the State.

A State department should provide up-to-date information. Bibliographies, news items, National Defense Education Act materials, and other such items are valuable helps in inservice improvement of teachers.

Practices on the State Level

The results of a questionnaire sent to State departments of education revealed that many do not have on their staffs supervisory personnel specially trained in mathematics, although others have added such supervisory persons within the past year.

Several States have developed, or are developing, programs of inservice education for mathematics teachers. For many decades New York State has had a staff of specialists in mathematics on the State level. This staff has been active in continuous curriculum revision, experimentation, publications, inservice training, and allied activities. For this reason New York's program will be described in considerable detail.

New York State

New York State for more than half a century has recognized the need for specialized personnel in mathematics on the State department of education staff. The State has also provided funds in the budget to be used specifically for teachers' inservice education. For example, on recommendation of the regents, the 1958 legislature appropriated $480,000.

For the secondary school mathematics and science teachers, funds from the 1958 appropriation were to be used to enhance their subject-matter backgrounds in order that they might (1) teach mathematics more competently in their respective courses, and (2) serve as resource teachers for their colleagues so that the latter also might improve their mathematics teaching. Two types of programs were covered by the funds, summer institutes and part-time courses during the school year; and both were to be given by appropriate institutions of higher education.

For the 1959 fiscal year the New York State Legislature appropriated $370,000 for refresher programs in mathematics and science for secondary school teachers of these subjects. An appropriation has been requested for 1960.

The funds appropriated were used to provide capable teachers with scholarships at summer institutes and academic-year courses. The summer scholarships, made available to both public and independent school teachers, comprised the following amounts:

Tuition and fees in full.
A $25 weekly allowance to resident participants toward room and board.
A $5 weekly allowance to commuting participants.

Eligibility requirements for both summer and academic-year programs were the following:

Applicants must state intent to continue teaching secondary school mathematics in New York State schools beyond the 1958–59 academic year.

Applicants must be accepted as graduate students by a New York State college or university offering approved programs.

Applicants must be recommended by their boards of education (superintendent or supervising principal).

Applicants must hold a baccalaureate degree.

Applicants must be properly certified.

A trial evaluation seems to indicate that the New York State programs generally have helped to bring the participants up to date and to improve their teaching. As to content material, it was geared to the level of the teachers' understanding, it was stressed, and it was new. As to followup reading in mathematical literature, the teachers have been stimulated to do more than previously.

The New York State Education Department puts out many publications of high quality. To find out what is available in mathematics write to the department, Albany 1.

Minnesota

The Minnesota inservice education program for both elementary and secondary teachers consists, in the main, of workshops established in local school systems or on a regional basis. The number of teachers (120+) in Minnesota who are participating in the School Mathematics Study Group through the Minnesota National Laboratory, an agency of the State department of education, has focused considerable activities on inservice education. The summer Laboratory institutes plus the National Science Foundation institutes have also had a great influence. The educational television station in Minneapolis has also provided help in inservice education.

New Mexico

The New Mexico program consists of a series of 1-day (Saturday 8 a.m. to 3 p.m.) workshops to which out-of-State consultants are invited. Teachers must pay their own expenses to attend these conferences.

The State department of education also makes available, on a loan basis, experimental materials such as those prepared by the School Mathematics Study Group at Yale, by the University of Maryland for grades 7 and 8, and by the University of Illinois Committee on School Mathematics.

Rhode Island

To stimulate interest and to help prepare periodicals in the teaching of mathematics and science, the Rhode Island State Legislature in 1959 established the Rhode Island Mathematics and Science Scholarship Commission and appropriated $30,000 annually for scholarships to be awarded by the Commission. Grants up to $500 annually may be made to eligible applicants. These grants are limited by law to:—

Juniors and seniors in college who are preparing to teach mathematics and/or science.

College graduates who are interested in preparing to teach mathematics and/or science.

Teachers in service who want to improve their teaching competence in mathematics and/or science.

Grants are made for study during either the regular school year or the summer.

In addition to this program, other State scholarship grants for teachers in service are administered by the State department of education, but are not necessarily limited to study in mathematics and science. The annual appropriation for these grants in recent years has been approximately $45,000. They are available for study at the two State-controlled and three privately controlled institutions of higher education within the State.

Pennsylvania

Under title III of the National Defense Education Act, Pennsylvania is carrying on inservice teacher education in the following manner.

With the approval of a State coordinator, population centers under the leadership of a local regional chairman may profit by an inservice education program for mathematics teachers. The regional chairman will first identify a group of at least 40 mathematics teachers desiring a program. Local colleges, universities, and industries are then surveyed for competent instructors. The content, as well as the length of the program, is determined by the interested teachers under the direction of the regional chairman. He then completes an application form and submits it to the State coordinator.

The coordinator studies the application with the assistance of a committee. If the application is not immediately approvable, suggestions are made for strengthening or improving it.

A significant feature of the programs is the fact that the entire cost of instructors' salaries is paid by evenly matched State and Federal funds. Local districts must provide a suitable meeting place for the class sessions.

Most programs range in length from ten to twenty-two 2-hour sessions. All of them center upon the explanation and development of modern mathematical concepts and experimental programs. Pennsylvania has available for use in its mathematics inservice programs one set of the New York State films titled, "Mathematics for Teachers," and has ordered a second set. However, an instructor need not use these films for his inservice program. In this case the instructor submits to the department for approval an outline of the material he plans to present.

No college credit can be earned by teachers through participation in these inservice programs. It is expected that teachers will attend regularly and respond actively to the assignments made by the instructor.

At the close of a program the regional chairman is responsible for giving to the State coordinator a statement of attendance at the sessions, an evaluation of the program's merit or success, and an indication of any possible weaknesses.

Contributions by the National Council of Teachers of Mathematics

One of the cooperating groups in the present conference is the National Council of Teachers of Mathematics, a professional organization with a membership of more than 37,000 elementary and secondary teachers and institutions. Each member of the NCTM receives either or both of the professional journals, *The Mathematics Teacher* or the *Arithmetic Teacher*. The former is devoted primarily to problems, practices, and professional articles on the teaching of secondary school mathematics, and the latter serves a similar function for elementary school mathematics.

Annually, the NCTM sponsors three or four professional meetings which attract mathematics teachers from all over the United States and many parts of Canada. Although it is difficult to measure the total impact of meetings on the reeducation of teachers, professional meetings such as those conducted by the NCTM do provide inspiration for teachers, spur many of them to additional independent study, and create a desire for additional education on college campuses.

Another valuable service of the NCTM is a series of publications. Their yearbooks, especially in recent years, have been directed toward the reeducation of the mathematics teacher. For example, the 23d, 24th, and 25th yearbooks are entitled, respectively, *Insights into Modern Mathematics*, *The Growth of Mathematical Ideas* (*Grades K–12*) and *Instruction in Arithmetic*. These publications are suitable for both independent and group study.

Supplementing the series of yearbooks, the NCTM publishes a number of pamphlets which may be classified as follows:

1. *Guide to the National Defense Education Act.* Advice on the use and procurement of teaching aids and tests, designs for the mathematics classroom, and the role of the mathematics supervisor.
2. *Curriculum and program.* Reports of curriculum committees, new developments in secondary school mathematics, the education of the gifted student in mathematics, and the education of the slow learner.
3. *"How-to Series."* How to use effectively, for mathematics teaching, the bulletin board, field trips, films and filmstrips, and the library; how to develop a teaching guide.
4. *Enrichment and miscellaneous.* Various topics to enrich high school mathematics teaching: Recreational mathematics, paper folding, the number concept, and geometry. One pamphlet on organizing and conducting a high school mathematics club.

The interest of local and State groups in helping their mathematics teachers improve themselves through inservice activities is indicated by a rapid growth in the program of the affiliated groups of the National Council of Teachers of Mathematics. The number of affiliated groups has grown to 75, an increase of about 15 during the last 5 years. A number of new groups are preparing to affiliate with the NCTM. Local and State groups have served their members through three main types of activities: Conferences and workshops, newsletters, and consultant services.

Every affiliated group conducts at least one conference per year. In a number of States, such as Illinois, a series of meetings is held in different geographical areas in order to serve better the needs of teachers throughout the entire State. The programs for many of these meetings present discussions of current and pressing problems by prominent mathematics educators. An outstanding example of a vital State conference is the Asilomar Conference sponsored each year by the California Mathematics Council. This meeting is a full-scale convention, with commercial and educational exhibits. Other conferences are of the workshop type, like those held in New England and Michigan. Several affiliated groups have cooperated with colleges in the sponsoring conferences and institutes, as in New Jersey and Florida.

Excellent articles on problems in mathematics teaching have appeared in the newsletters and journals published by the affiliated groups. Among the affiliated groups which produce newsletters or journals are these:

Association of Mathematics Teachers in New England.
Association of Mathematics Teachers in New Jersey.
Association of Mathematics Teachers of New York State.
California Mathematics Council.
Florida Council of Teachers of Mathematics.
Hillsborough County Mathematics Council, Florida.
Indiana Council of Teachers of Mathematics.
Kansas Association of Teachers of Mathematics.
Nebraska Section of the National Council of Teachers of Mathematics.
Ohio Council of Teachers of Mathematics.
Pennsylvania Council of Teachers of Mathematics.
Texas Council of Teachers of Mathematics.

Of special interest is the Workshop Consultant Service inaugurated in 1959 by the Ohio Council of Teachers of Mathematics which administers the service and subsidizes its administrative costs. The plan is one having two consultants for a 1-day workshop, each consultant to present in detail a different facet of the work of present curriculum study projects. The service is available to local school systems for a small fee. A major objective of the workshops is to motivate teachers to engage in independent study. The OCTM hopes that the consultant service can be expanded in future years into other areas of service. This pattern of operation of the OCTM might be adopted for use in other States.

Summary of Group Work-Session Deliberations

W. EUGENE FERGUSON

IN THE PAST, many good secondary mathematics programs, curriculum guides, courses of study, etc., have gathered dust on the shelves and caused very little, if any change in the mathematical content offered to secondary pupils. They have also had very little effect on the teaching of mathematics at the secondary school level.

Today it is different: it seems as though we are playing this game "for keeps" now. The groups that are developing new mathematics programs are, at the same time, striving to see that the people that use their material in its early stages of development have adequate training in the content and effective methods of presenting this mathematics.

The University of Illinois Committee on School Mathematics has been training teachers in the content and methods of teaching their program since the beginning in 1952. It was not until August 1959 that the the first course was made available to the public through the University of Illinois Press. The second, third, and fourth courses will become available in August 1960, 1961, and 1962, respectively. But the committee still strongly advises some study and training in content and method before using the material. The School Mathematics Study Group kept a rather tight control on the use of their materials grades 7–12 and set up centers for helping selected teachers with the content and methods of teaching this material during the year 1959–60. SMSG also very strongly advises teachers to study carefully the content of these courses before attempting to teach it to students.

The University of Maryland Mathematics Project and The Ball State Group have also recognized that they must accept some of the responsibility for mathematical reeducation of the teachers using their programs.

The Commission on Mathematics of the College Entrance Examination Board, in its report, recognized full well that any implementation of its program would require the reeducation of many of the mathematics teachers now in the classrooms.

It has become clear that some type of inservice education for mathematics teachers is an absolute necessity in all schools if we are to have a mathematics program that will meet the "Challenge of the Sixties."

It is just as obvious that all secondary schools do not need, and in fact cannot possibly use the same type of inservice education program.

The present Conference has had five work groups. On reporting a summary of their deliberations, I must admit immediately that I have selected many of them because they fit my own ideas as to what an inservice education program for mathematics teachers should be. Many of the topics and ideas I am presenting here were discussed in more than one group.

Mathematics teachers are human beings with feelings, and the majority want to do a good job in teaching mathematics and are willing to put forth considerable effort to get themselves prepared to teach mathematics that will meet the "Challenges of the Sixties." We have made teachers feel inadequate by too much loose talk about the introduction of "modern mathematics", "sets", etc., in the secondary school curriculum.

A much better approach would be to say to the teacher, "Lets upgrade the mathematics program: Here are some new materials that seem better fitted to meet the needs of pupils in the sixties. Here also are some of the ways that experienced teachers have found helpful in getting themselves prepared to teach an improved mathematics program." With this approach no teacher should feel threatened or develop an inferiority complex.

The first suggestion of an inservice mathematics education program for any school may be made by the principal, the mathematics supervisor, or the teachers themselves. No matter who suggests it first, it will be a happy event if it comes out of one or more mathematics teachers' meetings where the recommendations of the Commission on Mathematics, the School Mathematics Study Group Program, the University of Illinois Committee on School Mathematics Program, and other mathematics programs have been discussed. For an inservice education program to be a success the teachers must feel the need for it, or at least realize the need for it as the program unfolds. In one sense, this orientation process is inservice education, but in the next step the emphasis will be on the intensive study of specific courses, their content, and the best known teaching techniques.

The primary purpose of the inservice program is to enable teachers to conduct better mathematics courses and produce a more mathematically enlightened student body. That is, the real value of the inservice program is measured by what happens to the students, although I would not want to discount for one moment its effect on teachers. In general, they are happy and their enthusiasm runs high.

Continuous learning by the mathematics teacher is now necessary. This can happen through individual or group study.

One of the work groups listed these as characteristic of a good inservice program:

1. The inservice course must be taught in such a way that the teacher can translate it into classroom practice.
2. There must be financial support and released time for the teacher.
3. The financial support may come from private, governmental, State, county, or local funds.

A group listed the following types of activities as inservice education programs:

1. Courses or seminars offered for credit by the university or college professor, either oncampus or offcampus.
2. Courses taught by the schools themselves using their own staff, supervisors, teachers, etc.
3. Noncredit courses given by university professors.
4. National Science Foundation academic year institutes.
5. Fellowship for summer study offered by National Science Foundation, Esso Foundation, Shell Foundation, etc.
6. Departmental meetings.
7. One- and two-day conferences.
8. Informal study groups to work on specific topics such as (*a*) the place of proof in algebra, (*b*) level of rigor in geometry, (*c*) how much and what type of solid geometry should be taught, etc.
9. Workshops (1 or more weeks) where teachers study new materials and films, teach demonstration classes, and discuss procedures, etc.
10. Trips to computing centers.
11. Participation in National Council of Teachers of Mathematics and Mathematics Association of America activities.

The following guiding principles were listed by one group:

1. The central theme is "Mathematics and Its Teaching."
2. The mathematics teachers should play a vital part in structuring the program.
3. The program must have the genuine support of the administration.
4. The program requires careful and enlightened planning of activities.
5. Competencies of the individual teachers must be considered.
6. There should be a climate conducive to free interchange of ideas.
7. Evaluation of the program should be provided.
8. Books and materials should be made accessible to teachers.
9. Teachers' improved competence should be recognized in promotion and pay raises.
10. Incentives for participating in the program must be provided.

Operation Bootstrap is composed of activities and programs which local school systems may inaugurate and which do not require cooperation of any State or National organizations.

Here is a fruitful course of action suggested for a senior high school:

1. Select a single level of instruction for intensive study, such as 9th-grade algebra.

2. Select materials of instruction for this level as prepared by School Mathematics Study Group, University of Illinois Committee on School Mathematics, Ball State, etc. These materials should be studied by the entire mathematics staff.
3. Devote at least one semester and preferably 1 year to the study of the materials selected. The study sessions should be of approximately 2 hours' duration, at least once a week, and at a time convenient for all members of the mathematics staff. Each teacher should regard these sessions as an opportunity to understand the new materials in relation to the total mathematics program. This requires topic-by-topic discussion of the student textbook, solution of problems in the student textbook, tentative construction of tests and examinations, as well as a careful study of the related teachers' commentary.
4. Have a qualified consultant work with the mathematics staff during the preliminary study sessions and throughout the first year of presentation in the classroom.

On the assumption that the local community, its board of education, the school administrators, and the teachers of mathematics themselves feel and understand the need for inservice education, certain steps can profitably be taken. (May I say parenthetically that this is a tremendous assumption and I think it took about 2 years of quietly planting "idea germs" in the minds of the teachers and administrators before we in Newton were ready to move on an inservice program.)

Step 1.—Employ a coordinator to spark teacher interest, etc. As a *minimum* his mathematical preparation should include mastery of the mathematical concepts basic to implementing the programs of such nationally known groups as the Commission on Mathematics of the College Entrance Examination Board, School Mathematics Study Group, University of Illinois Committee on School Mathematics, and University of Maryland Mathematics Program.

Step 2.—Have a subject matter specialist conduct seminars dealing with content related to a specific secondary school course. This will tend to prevent teachers from acquiring only a little information about a lot of things.

Step 3.—Give teachers released time to observe a master teacher actually demonstrating with students how to teach the experimental course lessons. The most effective way to answer the often-asked question "Isn't this material too difficult for an eighth-grade class?" is to take such a class, teach the material, and let the questioner observe the results.

I dare say that if this type of inservice education program for grade 9 were followed by a similar one for grades 10, 11, and 12, there would be a decided upgrading of our product—a mathematically well-trained senior ready to enter college. If such a program were started in grade 7 for the mathematically talented, we would have many students able to take a full year of college calculus in their high school senior year. This naturally calls for an inservice course in the calculus for the teacher.

One group spelled out beautifully the responsibilities and opportunities of the superintendent, the supervisor, the principal, and the teacher. Here are a few details:

The superintendent:

1. Makes recommendations to the board of education regarding the development of a good mathematics program.
2. Provides financial support for special services necessary to implement the program, such as consultant services, attendance at conventions, etc.
3. Makes provisions for a good inservice program for mathematics teachers by allowing released time, supplying needed materials, etc.

The supervisor:

1. Informs the superintendent as to the existing mathematics program and current national trends.
2. Helps interpret the programs in mathematics to the public. (*This must never be neglected.*)
3. Keeps the mathematics teacher informed as to current literature, practices, and experimentation.

The principal:

1. Stimulates the professional growth of his teachers by encouraging them to participate in professional organizations, conventions, institutes, etc.
2. Helps interpret the programs in mathematics to the public.

The teacher:

1. Actively participates in inservice activities.
2. Affiliates with professional groups and activities concerned with the teaching of mathematics.
3. Shares experiences attained through experimentation, institute attendance, and individual study.

One group would add the following practices:

1. Have a master teacher conduct an experimental course. Lessons may be used as demonstrations for other teachers on released time.
2. Lengthen the teacher's contract "year," of course with the proper pay increase. The additional time could be used for curriculum or inservice training.
3. Send (with local funds) selected teachers to participate in seminars and workshops conducted by such groups as School Mathematics Study Group, University of Illinois Committee on School Mathematics, University of Maryland Mathematics Program, etc. These teachers could later be used as group leaders in local study programs.

The principal responsibility for providing an inservice program rests with the local school system.

One of the working groups offered the following pointers:

1. Local support is basic to a program's success. An effective method is to interest local industries to help the school by supplying expert assistance, finacial aid, or even both.

2. Sabbatical leaves with full pay or half-pay (so that a teacher may attend an academic year institute without financial loss) is a practice that should be more widespread.
3. An adequate professional mathematics library for the use of students and teachers is a necessity today. Good book lists are readily available.
4. A visit by a teacher to another school where experimental projects in mathematics are being taught is an excellent way to excite his imagination.
5. State departments of education can (1) hold regional and statewide meetings on the content of the new mathematics, (2) provide consultant services to the schools, and (3) provide leadership in curriculum development.

It is clear from the work of the five groups that nearly all schools can have a creditable inservice training program if the teachers and administration see the need for one.

To keep inservice education programs operating year in and year out, as in Philadelphia and other places, there must be strong leadership by the administration and some members of the teaching staff.

The theme for the Department of Classroom Teachers of the National Education Association for this year is "Time to Teach." I feel that after this working conference, the conscientious mathematics teachers could very well adopt the theme "Time To Teach and Time To Study."

Panel Discussions

Chairmen for the Discussions:
DR. MILTON BECKMAN
DR. CLARENCE B. LINDQUIST

Panel Members:
DR. W. L. DUREN, JR.
DR. HENRY SYER
DR. HENRY VAN ENGEN

The method for conducting the panel discussions was for members of the conference to ask questions to be answered by members of the panel. Selected questions and answers follow below:

Q. *Should certification requirements for teachers be raised? What would be the effect on the supply of teachers?*

A. This conference is attempting to answer just those questions. We shall have to put into active teaching those who cannot fully meet certification requirements and those who can. Then we shall have to raise standards by means of inservice programs such as those being considered by this Conference. This can be done by (*a*) raising the teachers' qualifications step-by-step until they meet certification standards, and (*b*) making salary increases dependent on increased qualifications. This recommendation applies to teachers now in service and to those yet to be employed.

Q. *Where are training personnel to be found to do the work of guiding teachers through the steps of improvement?*

A. The resources are mainly in the colleges. A way must be found for increasing the number of persons who can teach college mathematics and can also help in the program of training high school teachers. Probably many of these persons will have to come from the present members of high school faculties.

A. To meet the teacher-shortage problem, both at the high school level and at the college level, teachers will have to handle larger numbers of students in all classes, perhaps two or three times as many as at present.

A. There are two sources of personnel to meet the increased demands: (1) Retired military persons who can be trained as teachers; (2) mothers of families whose children are grown and who have enough background and educational experience to be upgraded through the school's inservice education facilities.

A. Put gifted students through a brief training course and then have them take over a class under supervision. This suggestion would apply in the colleges. Could it also apply in high schools?

Q. *How can students, parents, and teachers be made to realize that there is a very real emergency?*

A. To this question there seems to be no satisfactory answer. However, realization of the existence of an emergency may be more widespread than apparent.

Q. *What use can be made of teaching machines?*

A. Two psychologists are at present studying the School Mathematics Study Group materials and will select units that can be programed for use with the teaching machines. Such machines seem to be working satisfactorily in certain colleges and high schools. They should help teachers as well as students.

Q. *What efforts are being made to get more persons to take up teaching as a career?*

A. This is an important question but its answer is not the concern of the present conference. This conference was called to consider the one problem of inservice education.

Q. *How can television and tape recordings help solve the problem of inservice improvement?*

A. Through having master teachers prepare and teach model lessons on TV there will be a gain in subject-matter mastery, and improved teaching methods. Such programs can be recorded and made available for repeated use. Teachers as well as students will profit.

Q. *Have books lost their place as effective aids to inservice improvement?*

A. No. A good book is still, and will remain, one of the most effective aids to improvement.

Q. *Does it always follow that fewer persons go into teaching when certification requirements are increased?*

A. No. For example, Kansas increased their certification requirements, and found that the number of available teachers actually increased.

Q. *Is teacher education being done more poorly today than formerly?*

A. No. Of those who are now being reeducated, an ever increasing number are being taken by industry, so the programs must have been satisfactory.

Q. *What would be the effect of offering good teachers substantial monetary inducements?*

A. It would depend on the number of such prizes to be offered. If there were only a few, the general effect would be insignificant.

A. If we educate teachers more fully than at present and give no adequate compensation in salary, we shall find that we are educating them right out of their jobs, because industry will take them.

Q. *Are special inservice education programs the only answer to the problem of teacher improvement?*

A. No. The problem might be solved better by giving each teacher $75 a month to attend regular oncampus graduate classes.

Q. *Is the problem of inservice education necessarily one that must be solved at the national level?*

A. No. It should be solved at the local level. Local systems should not wait for a national solution but should organize their own programs. As a part of the solution of the improvement problem, teachers' salaries should be raised sufficiently to compete with those offered by industry.

Q. *Are the institutes reaching the persons who are most in need of improvement?*

A. The institutes often educate teachers who are least in need of such attention. One means of rectifying this situation is to have local teachers instructed by those who have had the benefits of institute attendance. The instructors should be paid for this extra workload.

Q. *What provisions need to be made for educating teachers to teach the slow learners?*

A. There should be experimentation with the University of Illinois Committee on School Mathematics materials in an attempt to make adjustments to the needs and interests of slow learners. The School Mathematics Study Group also is considering the problem seriously. One aspect is that of finding more interesting material for the slow learners.

Implications of the Conference

OBVIOUSLY a conference on inservice education of secondary school mathematics teachers has wide implications. Although it is of chief concern to these teachers and their supervisors, it is important also to mathematics teachers at other educational levels and to secondary school teachers of other subjects. It has implications for mathematics education at the preservice level as well as at the inservice level.

The concluding session of this conference provided an opportunity for a panel of specialists representing related interest areas to express their reactions to various aspects of the conference. The interest areas and the specialists were the following:

THE ARTS	Dr. Chester L. Neudling, Specialist for the Humanities, U.S. Office of Education.
COLLEGE MATHEMATICS	Dr. Leon Cohen, Head of Mathematics Department, University of Maryland.
THE SCIENCES	Dr. Robert H. Carleton, Executive Secretary, National Science Teachers Association, National Education Association.
SCHOOL ADMINISTRATION	Mr. Carl L. Fromuth, Superintendent of Philadelphia District No. 1 Schools.
TEACHER EDUCATION	Dr. Robert Poppendieck (*Chairman of the panel*), Specialist for Teacher Education, U.S. Office of Education.

The discussion was informal and provided for reactions, both to the conference in general, and to specific ideas developing during the conference. It provided for points of commendation, supplemental suggestions, caution about difficulties, and suggestions on implementation. The key questions used were the following:

1. **Which proposals for advancing the local district reeducation of high school mathematics teachers seem to you to be most promising?**
2. **Which proposals have the most provocative long-range values?**
3. **What stumbling blocks do you see that districts and inservice reeducation sponsors should be cautioned about?**
4. **What elementary-school, and what college-level, articulation factors should be considered?**
5. **What are the implications for revision of preservice education for mathematics teachers?**
6. **What are the implications for the inservice reeducation of college mathematics teachers? For teachers in other disciplines? For teacher education generally?**

Discussion centered on four major concerns: (1) the communication of conference materials and development of better mutual understanding between specialists in mathematics and those in other areas, (2) the comprehensive nature of inservice responsibilities and activities, (3) the importance of effective articulation between levels as well as disciplines, and (4) clarification of objectives.

Communication and Mutual Understanding

In providing both summary and detailed reports on the conference, teachers in disciplines other than mathematics must be taken into account. New concepts in mathematics are the concern of all people and, hence, of all teachers. The approach must be a human one for some may be sensitive about the recent emphasis and opportunity for mathematics. The central role of mathematics must be set forth. Its development and advance must be paralleled by refinements in other disciplines as well. Mutual efforts among the disciplines are required both in moving forward the frontiers of knowledge and in reeducating teachers in all pertinent areas.

The importance of cooperative activity in achieving mutual understanding must not be underestimated. Mathematics, in a sense, is a universal language—among disciplines as well as among peoples. Nevertheless, competence in comprehending and employing it falls far short of its importance. Teachers generally must have a part in planning to study the new concepts in mathematics, if real progress is to be made. This in turn will support the activities of other disciplines as they look to their horizons. Participation rather than specialization is the key to advancing the frontiers of knowledge as required by these times.

Comprehensive Nature of Responsibilities and Activities

Inservice programs generally are developed comprehensively rather than for separate features. Many classroom activities focusing on pupil experiences cut across disciplinary lines. Field and extension services are being capitalized. Awards programs for teachers are appearing. The public is being increasingly involved, and public interest is high. Increasing attention needs to be given to cooperative inservice programs developed by faculties through interdisciplinary cooperation and making the greatest use of total institutional resources.

Indeed, it may well be that wide participation in comprehensive planning of inservice education and professional development is the very means of insuring the specific attention of all faculty members to the urgency and the opportunity.

Articulation

Of primary importance is articulation between individual high schools and the colleges and universities that accept their graduates. This involves face-to-face contacts and the development of mutual understanding. Professional rapport is to be established only through deliberate and careful cooperative effort. By extension, then, more effective articulation needs to be developed relating the basic work in elementary school to that in both high school and college.

Elementary mathematics has been neglected. The problem at this level now exceeds that in the secondary schools. If effective progress is to be made in the secondary schools and colleges, attention must be given to the general problem of mathematics in the elementary school, to the mathematics preparation of general teachers, to the preparation of upper-grade specialists, and to slow learners in arithmetic. Although articulation generally has both horizontal and vertical relationships, in mathematics it is crucial.

Clarification of Objectives

Attention to articulation brings into focus more specific attention to the objectives. Are computational skills, only, to be sought in the schools, or must rich meanings be established? The implication for more precise and far-reaching goal definition is clear. Richer meanings lead to sharper skills, better research, and more effective applications. As frontiers of knowledge move forward and as patterns of living become increasingly complex, it is on sound and basic meanings that we must rely rather than on simple and ever-changing techniques.

Reeducation, then, and continuing professional development are essential. Growth must vary with individuals and with circumstance. Responsibility for inservice reeducation is, and must remain, local. Education is a profession, and continuing growth is a critical professional element. As educators at their various levels and in their several disciplines discharge their specialized responsibilities through inservice reeducation, this must be maintained. It is highly important that each professional educator assume the full stature of professional educator. This is more than a truism: it is a principle. From demonstrated professional action, effective reeducation flourishes.

As mathematics teachers, supervisors, and professors act to bring up to date and keep up to date the mathematical competencies of high school teachers, the example may well carry over to other disciplines. The techniques of institutional programs, workshops, small-district cooperation, university contract, leadership stimulation, and personal programs of continuing professional study—these may all be well imitated. Of obvious carryover is the inservice professional development of mathematics staff members in the colleges and universities. Their effective and continuing updating is prerequisite to effective programs in inservice and preservice mathematics programs for their students. Science teachers, of course, are immediately involved in the mathematics programs and should carry the impact over to reeducation programs in science content. Less obvious, perhaps, but of growing interest and concern are the implications for the humanities. Modern languages seem to be leading the way in reeducation here, but such is the interdependent nature of our culture that no discipline can lag without damage to all. All disciplines may profit by examining the implications to themselves of the reeducation emphasis in mathematics. Possibly the most crucial is teacher education, itself. Not only in professional education, but in all education, the updating and continuing improvement of the teacher and the professor in the understandings and competencies of his profession of teaching is paramount. As balance in today's way of life turns on brainpower, so the optimum achievement of that brainpower turns on the teaching competencies of the faculties of the Nation's schools and colleges. In the effective directing of learning, there is no substitute for quality; there is no substitute for growth.

Appendixes

A—E: Supplementary Lists—Address by Henry W. Syer

F: Participants in the Conference

G: Members of the Conference Planning Committee

Appendix A

Helpful Books, Curricula, and Programs Reported by State Departments of Education

Books

Major Emphasis: Content

Elementary School Level

Hogben, L. T. *Wonderful World of Mathematics.* New York: Doubleday (Garden City), 1955.

Meyer, J. S. *Fun With Mathematics.* Cleveland: World Publishing Co., 1952.

Secondary School Level

Adler, Irving. *The New Mathematics.* New York: Day, 1958.

Aiken, D. J. *and* C. A. Beseman. *Modern Mathematics: Topics and Problems.* York: McGraw-Hill, 1955.

Allendoerfer, C. B. *and* C. O. Oakley. *Principles of Mathematics.* New York: McGraw-Hill, 1955.

Commission on Mathematics, CEEB. *Introductory Probability and Statistical Inference.* College Entrance Examination Board. New York, 1959.

Kemeney, J. G., J. L. Snell *and* G. L. Thompson. *Introduction to Finite Mathematics.* Englewood Cliffs, N.J.: Prentice-Hall, 1957.

Newman, J. R., *ed.* *The World of Mathematics.* New York: Simon and Shuster, 1956.

Stabler, E. R. *Introduction to Mathematical Thought.* Reading, Mass.: Addison-Wesley, 1953.

Major Emphasis: Method

Elementary School Level

Spitzer, H. F. *Practical Classroom Procedures for Enriching Arithmetic.* St. Louis, Mo.: Webster, 1956.

Secondary School Level

None.

Curricula and Programs

Various curricula and programs named by State departments of education as helpful appear in the list below, which indicates also the number of times each item was named:

Item	*Number of times named*
School Mathematics Study Group	10
Commission of CEEB	5
University of Illinois Committee on School Mathematics	5
University of Maryland Mathematics Project	3
Ball State Teachers College materials	2
Boston College Materials	1
New York Television Project	1
Other Materials	1

Miscellaneous

Miscellaneous helpful aids named by State departments are listed below with the number of times each was named:

Item	*Number of times named*
Films	3
Publications of the National Council of Teachers of Mathematics (NCTM)	2
23d Yearbook of NCTM	4
24th Yearbook of NCTM	6
The Mathematics Teacher	2
The Arithmetic Teacher	1
The Mathematics Student Journal	1
Conant's Report on the American High School	1

Appendix B

Helpful Books Reported by School Systems

Major Emphasis: Content

Elementary School Level

Buckingham, B. R. *Elementary Arithmetic.* Boston: Ginn, 1953.

Larsen, H. D. *Arithmetic for Colleges.* New York: Macmillan, 1958.

Mueller, F. *Arithmetic: Its Structure and Concepts.* Englewood Cliffs, N.J.: Prentice Hall, 1956.

Swain, R. L. *Undertaking Arithmetic.* New York: Rinehart, 1957.

Secondary School Level

Adler, Irving. *The New Mathematics.* New York: Day, 1958.

Aiken, D. J. *and* C. A. Besemann. *Modern Mathematics: Topics and Problems.* New York: McGraw-Hill, 1959.

Allendoerfer, C. B. *and* C. O. Oakley. *Principles of Mathematics.* New York: McGraw-Hill, 1955.

Andree, R. V. *Selections From Modern Abstract Algebra.* New York: Holt, 1958.

Bell, E. T. *Men of Mathematics.* New York: Simon and Schuster, 1937.

Birkhoff, G. *and* S. MacLane. *Survey of Modern Algebra.* New York: Macmillan, 1953.

Breuer, J. *Introduction to the Theory of Sets.* Englewood Cliffs, N. J.: Prentice-Hall, 1958.

Brixey, J. C. *and* R. V. Andree. *Fundamentals of College Mathematics.* New York: Holt, 1954.

Courant, R. *and* H. Robbins. *What Is Mathematics?* Fairlawn, N.J.: Oxford, 1941.

Dantzig, T. *Number: The Language of Science.* New York: Macmillan, 1954.

Eves, H. *and* C. V. Newsom. *Introduction to the Foundations and Fundamental Concepts of Mathematics.* New York: Rinehart, 1957.

Fehr, H. F. *Secondary Mathematics.* Boston: Heath, 1951.

Fisher, R. C. *and* A. D. Ziebur. *Integrated Algebra and Trigonometry.* Englewood Cliffs, N.J.: Prentice-Hall, 1958.

Freund, J. E. *A Modern Introduction to Mathematics.* Englewood Cliffs, N.J.: Prentice-Hall, 1956.

Kameney, J. G., J. L. Snell *and* G. L. Thompson. *Introduction to Finite Mathematics.* Englewood Cliffs, N.J.: Prentice-Hall, 1957.

Kline, M. *Mathematics in Western Culture.* Fairlawn, N.J.: Oxford, 1953.

Maria, M. H. *Structure of Arithmetic and Algebra.* New York: Wiley, 1958.

May, K. O. *Elements of Modern Mathematics.* Reading, Mass.: Addison-Wesley, 1959.

Meserve, B. E. *Fundamental Concepts of Algebra.* Reading, Mass.: Addison-Wesley, 1953.

——— *Fundamental Concepts of Geometry.* Reading, Mass.: Addison-Wesley, 1955.

Newman, J. R., *ed.* *The World of Mathematics.* New York: Simon and Schuster, 1956.

Polya, G. *How To Solve It.* New York: Doubleday, 1957.

——— *Mathematics and Plausible Reasoning.* Princeton, N.J.: Princeton University Press, 1954.

Richardson, M. *Fundamentals of Mathematics.* New York: Macmillan, 1958.

Stein, E. I. *Fundamentals of Mathematics.* Boston: Allyn and Bacon, 1952.

Vance, E. P. *Unified Algebra and Trigonometry.* Reading: Addison-Wesley, 1955.

Tarkski, A. *Introduction to Logic and the Methodology of Deductive Science.* Fairlawn, N.J.: Oxford, 1946.

Woodward, Edith J. *and* Roderick C. McLennon. *Elementary Concepts of Sets.* New York: Holt, 1959.

Major Emphasis: Method

Elementary School Level

Banks, J. H. *Learning and Teaching Arithmetic.* Boston: Allyn and Bacon, 1959.

Brueckner, L. J. *and* F. E. Grossnickle. *How To Make Arithmetic Meaningful.* Philadelphia: Winston, 1947.

Clark, J. R. *and* L. K. Eads. *Guiding Arithmetic Learnings.* New York: World Book Co., 1954.

Gattegno, C. *Child's Conception of Number.* Routledge: British Book Service, 1953.

Morton, R. L. *Teaching Children Arithmetic: Primary, Intermediate, Upper Grades.* Morristown, N.J.: Silver, 1953.

Secondary School Level

National Council of Teachers of Mathematics. *Mathematics for Academically Talented Students.* Washington: NEA.

Miscellaneous

Miscellaneous helpful aids named by school systems are listed below with the number of times each was named:

Item	*Number of times named*
Yearbooks of NCTM	15
The 16th	1
The 18th	1
The 23d	12
The 24th	17

OTHER PUBLICATIONS

The Mathematics Teacher	14
The Arithmetic Teacher	7
American Mathematical Monthly	1
Scripta Mathematica	1
Mathematics Magazine	1
The Mathematics Student Journal	2
Scientific American	1
School Science and Mathematics	1

Appendix C

Helpful Sources of Curricular and Teaching Materials Reported by School Systems

Various sources of curricula and teaching materials reported by school systems, together with the number of times each was named, are listed below:

Item	*Number of times named*
School Mathematics Study Group	55
Commission of CEEB	51
University of Illinois Committee on School Mathematics	40
University of Maryland Mathematics Project	32
Madison Project, Syracuse University	7
Boston College materials	6
Other materials	5
Advanced placement bulletins	3
Ball State Teachers College materials	2
California geometry	1
	103

Appendix D

Mathematical Topics Studied by School Systems

Mathematical topics studied by school systems and the number of times each was named are shown below:

Arithmetic

Item	*Number of times named*	*Item*	*Number of times named*
Number systems	9	Place value	1
Number bases	2	Primes and factors	1
Systems of numeration	2	Principles of arithmetic	1
History of numbers	1		

Algebra

Inequalities	5	Field theory	3
Real number system	2	Matrix algebra	3
Advanced algebra	1	Group theory	2
Complex number system	1	Number theory	2
Order relations	1	Modulo number systems	1
Proof in algebra	1		

Geometry

Plane and solid geometry	8	Nonmetric geometry	1
Non-Euclidean geometry	4		

Other Topics

Probability and statistics	8	Elementary functions	1
Coordinate geometry	5	Trigonometry	1
Calculus	2	Sets	17
Digital computors	2	Logic	9
Limits	2	Variables, functions and relations	6
Applied mathematics	1	Finite mathematical systems	2
Calculus and analytic geometry	1		
Circular functions	1		

Appendix E

Mathematical Topics Considered Most Helpful by School Systems

Mathematical topics considered most helpful by school systems and the number of times each topic was named are shown below:

Arithmetic

Item	*Number of times named*	*Item*	*Number of times named*
Number systems	8	Structures of arithmetic	2
Number bases	2		

Algebra

Inequalities	3	Matrix algebra	1
Structures of algebra	2	Number fields	1
Groups and fields	1	Number theory	1

Geometry

Integration of plane and solid	1	Syllogisms in geometry	1
New definitions in geometry	1		

Other Topics

Probability and statistics	4	Logic	3
Computors	1	Functions	2
Coordinate geometry	1	Inductive and deductive systems	1
Sets	10		

Appendix F

Participants in the Conference

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Education
National Science Foundation
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Appendix G

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The following participants in the conference served as members of the planning committee: Myrl Ahrendt, Milton Beckman, Kenneth E. Brown, Clarence B. Lindquist, Robert Poppendieck, and Daniel W. Snader.